Easy 3D Phase Unwrapping

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INTRODUCTION

We present a new, robust technique for multidimensional phase unwrapping which is very simple to implement and to automate. Image phase maps are used in MRI for a number of reasons but the driving application for this work is EPI undistortion. An EPI image can be corrected based on an acquired map of the magnetic field, as described by Jezzard and Balaban [1], and others. Additional applications include automated shimming procedures, chemical shift imaging, temperature mapping, and phase contrast angiography. Calculated phase maps are correct to within additive factors of $2\pi n$, and need to be unwrapped across all imaged dimensions. The lower the SNR of the image, the more challenging this problem becomes. The phase unwrapping problem remains an area of active research, e.g. Refs. [2,3].

DATA ACQUISITION

The data which was necessary to compute the field-map was sampled using dual-echo gradient-echo, preserving the complex data. Discontinuities occur in places where the phase grows over π or under $-\pi$. Before unwrapping, automatic thresholding of the image is performed because in pixels that are outside of the imaged object, there is only noise, and the phase angle has no meaning.

UNWRAPPING METHOD

Initially, the thresholded phase image is divided into separate sets, so that within each set there are no 2π discontinuities. An algorithm for doing so is described below:

1. Choose a pixel X in the image that does not belong to any set - this is the seed of a new set, S_i.
2. Each neighbor, Y, of a pixel X in S_i, will be incorporated into S_i if it satisfies both of the following conditions:
    a. It does not belong to any set.
    b. The difference between Y and X is less than a continuity value (this value can be much smaller than π).
3. For every new member of the set S_i, repeat steps 2 and 3, until the set stops growing.
4. Stop if all the points in the phase image belong to one of the sets, otherwise go to step 1.

At the end of this stage of the algorithm, every point belongs to a set, and within each set there is continuity of phase.

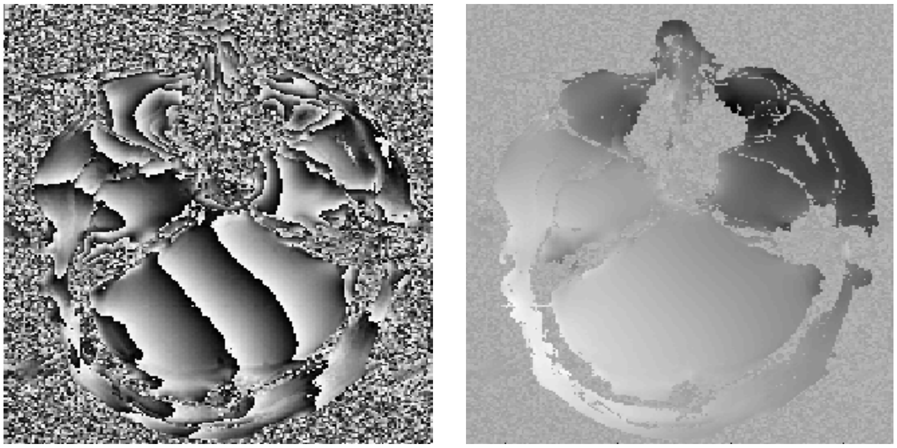
The second stage of the phase unwrapping algorithm is to merge the sets. This can be done by starting from a set S_1, and doing a "connect-procedure" between this set and each of its neighbors as described below:

1. Choose a neighboring set, S_2, to S_1. Find all the pairs $\{x_1,x_2\}$ such that x_1 is in S_1, x_2 is in S_2, and x_1 and x_2 are neighbors. For each such pair, denote the difference between the points as $d(x_1,x_2)$.
2. Find the minimum of $\{E_0 , E_+ , E_-\}$, where: $$\begin{cases} E_0 = \sum d(x_1, x_2)^2 \\ E_+ = \sum d(x_1,\ x_2+2\pi)^2 \\ E_- = \sum d(x_1,\ x_2-2\pi)^2 \end{cases}$$
3. If the minimum is E_+, increase the points in S_2 by 2π. If the minimum is E_-, decrease the points in S_2 by 2π.
4. Repeat steps 2 and 3 until E_0 is the minimum.
5. Combine S_1 and S_2, designate the new set S_1, and repeat the process from step 1.

The outcome of this process is one continuous set which contains all the pixels of the image.

RESULTS

A result for a 2D example is presented in Figure 1. One can see that, after executing the algorithm on a problematic slice, no 2π-discontinuities remain.



Left: Original phase-image. *Right*: Phase corrected image.

Figure 1: An example of results from the unwrapping technique.

SUMMARY

A new, conceptually simple, and inherently multi-dimensional algorithm for stable and robust phase unwrapping is described and demonstrated. The phase unwrapping technique successfully deals with noise and with areas of minimal connectivity. Moreover, 3D implementation would improve the results by adding additional information from adjacent slices. Our Matlab code is available upon request.

REFERENCES

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