Effects of (k, t)-space Sampling in Dynamic MRI

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Introduction: Dynamic imaging in MR is necessary for various applications such as cardiac and functional brain imaging. However, relationships and trade-offs between various dynamic MRI (DMRI) methods, and the sources of their image artifacts and measurement errors are not well-described with a conventional 2D (kx, ky)-space analysis. Previously, a (k, t)-space analysis was introduced to give better understanding of DMRI methods with Cartesian trajectories [1]. In this work, we expand on the 3D (kx, ky, t)-space analysis to provide insights into DMRI with arbitrary k-space sampling trajectories.

Theory: The reciprocal (or Fourier-transformed) space of (kx, ky, t) is denoted as (x, y, f), where f is temporal frequency, and 3D-FOV represents a non-aliased 3D center region in (x, y, f). Motion artifacts appearing in images displayed in (x, y, t)-space are due to aliasing occurring in 3D (x, y, f)-space [1]. These motion-induced aliasing artifacts arise because one cannot sufficiently sample along the t-direction at all (kx, ky)-points for given object dynamics, e.g., cardiac motion. To understand the effects of arbitrary k-space trajectories on motion artifacts, we utilize the following two general sampling principles: (a) Samples more evenly distributed, or isotropically spread out, within the sampling region reduce the intensity of aliasing by spreading it out, preventing concentrated high-intensity aliasing. (b) The amount of aliasing energy depends on the signal energy in the under-sampled region; previously, this reasoning justified the use of under-sampled variable-density trajectories in 2D (kx, ky)-space [2], where the under-sampled region is the outer 2D (kx, ky)-space of low signal energy. In the following, we give two examples of our 3D analysis that provides further insights into DMRI.

Analysis-1: A 3D sampling pattern of conventional Cartesian k-space trajectories can cause more aliasing along a certain direction in the 3D reciprocal-space (x, y, f), e.g., a direction corresponding to “3” in Fig. 1-a. In contrast, multi-shot spiral trajectories can distribute aliasing artifacts more diffusely compared to conventional Cartesian trajectories because of more isotropic sampling in 3D (kx, ky, t)-space (Fig. 1-b). In 3D (kx, ky, t)-space, most signal energy is concentrated around the (kx=0, ky=0, t)-line. Centric-sampling trajectories result in denser sampling around the (kx=0, ky=0, t)-line compared to non-centric ones, and thus motion artifacts caused by non-centric trajectories are likely to be of higher energy compared to artifacts caused by centric ones. Note that based on the principles (a) and (b), one can also see that interleaving or pseudo-randomizing the phase-encode ordering (e.g., Fig. 1-c) improves the temporal quality of DMRI [3,4] by sampling more isotropically overall and more densely sampling the (kx=0, ky=0, t)-line, where high signal energy exists.

Analysis-2: With a 2D (kx, ky)-space perspective, the superior motion-artifact suppression of a centric-sampling trajectory is often loosely attributed to “sampling the k-space origin at every TR” [5]. However, this 2D k-space analysis does not fully explain why a centric-sampling trajectory with pseudo-randomized ordering can produce better motion-artifact suppression than one with sequential ordering [6,7] although both sample the 2D (kx, ky)-space origin at every TR. With the 3D (kx, ky, t)-space perspective, one can see that pseudo-randomized ordering leads to more isotropic 3D-sampling, especially in the region around the (kx=0, ky=0, t)-line (compare Fig. 1-d with e), resulting in better motion-artifact suppression.

Discussion: This 3D perspective can be used to understand the trade-off between temporal and spatial resolution, which mainly depends on the different coverage and density of 3D-samples in (kx, ky, t)-space (Fig. 1-f). By using the k-space trajectories that are (a) more isotropic and (b) denser around the (kx=0, ky=0, t)-line, reduced-FOV imaging [3] is possible. The 3D isotropy can be quantitatively measured using the isoperimetric theorem on 3D voronoi cells of the sampling points (Fig. 2). The isoperimetric theorem in 3D states that a ball gives the maximum volume for a given surface area [8], which can be used to calculate the isotropy of a 3D volume.

Conclusion: A 3D sampling pattern that is more isotropic in (kx, ky, t)-space and denser around the (kx=0, ky=0, t)-line is likely to minimize overall motion-induced-aliasing artifacts in the 3D reciprocal-space (x, y, f), resulting in improved temporal qualities of DMRI. The presented sampling perspective in 3D (kx, ky, t)- and (x, y, f)-space can be used to appreciate and further improve DMRI methods by understanding their fundamental limits given by k-space trajectories.

References: [1] Xiang QS, et al., MRM, 29:422-428, 1993, [2] Tsai CM, et al., MRM, 43:452-458, 2000, [3] Madore B, et al., MRM, 42:813-828, 1999, [4] Tsao J, et al., MRM, 50:1031-1042, [5] Park JB, et al., MRM, 49:322-328, 2003, [6] Shankaranarayanan A, et al., JMRI, 13:142-151, 2001, [7] Pipe JG, et al., MRM, 41:417-422, 1999, [8] Chavel I, Cambridge University Press, Cambridge 2001

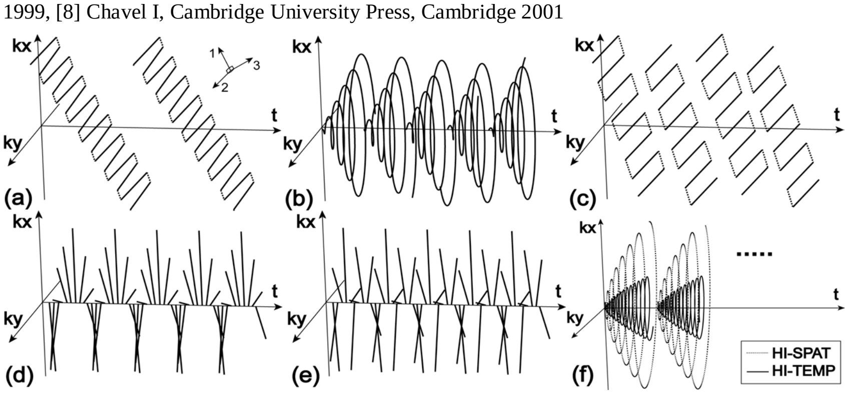

Fig. 1: (a) Cartesian, (b) spiral, (c) interleaved (even-odd) Cartesian, (d) sequential and (e) interleaved PR sampling in 3D (kx, ky, t)-space. (f) shows 16 interleave high spatial (HI-SPAT) and 4 interleave high temporal (HI-TEMP) resolution trajectories.

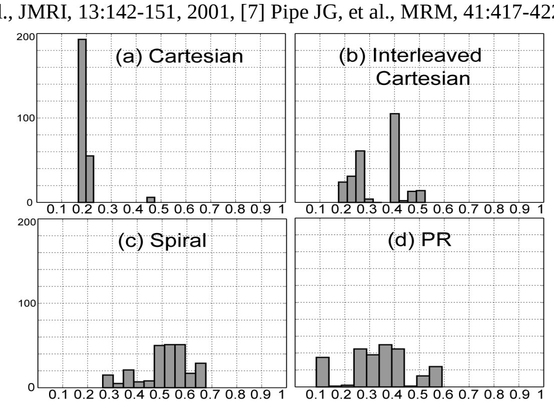

Fig. 2: 3D isotropy histogram of sampling points in 3D (kx, ky, t)-space. The horizontal axis represents isoperimetric ratio (IP). Higher IP indicates higher 3D isotropy.