# Rapid T1 Mapping by Variable Flip Angles: Analytic Expression for B1-error Influences and Optimization for Large T1 Range 

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## INTRODUCTION

Quantification of $T_{1}$ relaxation time is valuable in dynamic contrast-enhanced studies of cancer and for enhanced discrimination of other diseases, such as epilepsy and Parkinson's. To be clinically useful, the imaging time must be short to allow for time-resolved $T_{1}$ measurements over a large volume. The measurements must also be accurate over a large range of $T_{1}$ 's. Previous work (1,2) shows that the variable flip-angle (VFA) method achieves an accuracy similar to conventional inversionand saturation-recovery techniques but with a significant reduction in imaging time. It is also shown that only two ideal angles are required. However, the precision is optimized only over a narrow range of $T_{1}$ values, and the sensitivity to errors in flip angle, although acknowledged (3), has not been examined in detail.

In this study, the influence of errors in flip angle is developed theoretically and verified with numerical simulations. The accuracy and efficiency of $T_{1}$ measurements over a wide range of $T_{1}$ values are investigated using Monte Carlo simulations for different combinations of flip angles and are verified in phantoms.

## THEORY

The signal time course of a SPGR experiment is a function of $T_{1}, T R$, flip angle $\alpha_{i}$, and spin density $M_{\mathrm{o}}$, and is given by Eq.[1], where $E_{1}=\exp \left(-T R / T_{1}\right)$. The graph of $S_{i}$ versus $\alpha_{i}$ can be linearized according to Eq.[2], and $T_{1}$ can be extracted from the slope $E_{1}(1)$ :

$$
S_{i}=M_{o} \sin \alpha_{i}\left(1-E_{1}\right) /\left(1-E_{1} \cos \alpha_{i}\right) \quad[1] \quad\left(S_{i} / \sin \alpha_{i}\right)=E_{1}\left(S_{i} / \tan \alpha_{i}\right)+M_{o}\left(1-E_{1}\right)
$$

The error in $T_{1}\left(d T_{1}\right)$ due to errors in flip angle $\left(d \alpha_{i}\right)$ can be derived by propagating $d \alpha_{i}$ through the ordinates and slope of Eq. [2] into $T_{1}$. For small $d \alpha_{i}$, the error in $T_{1}$ is:

$$
\begin{equation*}
d T_{1}=\frac{-T_{1}^{2} e^{T R / T_{1}}}{T R \cdot N\left(\overline{X^{2}}-\bar{X}^{2}\right)} \sum_{i=1}^{N} d \alpha_{i}\left(1 / \tan \alpha_{i}\right)\left\{Y_{i}\left(X_{i}-\bar{X}\right)+X_{i}\left(1+\tan ^{2} \alpha_{i}\right)\left(Y_{i}-\bar{Y}-2 e^{-T R / T_{i}}\left(X_{i}-\bar{X}\right)\right)\right\} \tag{3}
\end{equation*}
$$

where $N$ is the number of flip angles, $X_{i}=S_{i} / \tan \alpha_{i}, Y_{i}=S_{i} / \sin \alpha_{i}$, and $\bar{X}$ and $\bar{Y}$ represent the mean values.

## METHODS

Simulations were performed by generating signal data for $T_{1}=200-3000 \mathrm{~ms}$ using Eq.[1]. Pairs of angles optimized for single $T_{1}$ values (1) in this range were considered, as were combinations of three or more of these angles. Precision was assessed by corrupting the signal with Gaussian noise ( $\mu=0$, $\sigma=0.002 M_{\mathrm{o}}$ ), estimating $T_{1}$ from Eq.[2], and computing $\mu_{\mathrm{T} 1}$ and $\sigma_{\mathrm{T} 1}$ from 65,000 independent trials. Different parameter settings were compared on the basis of efficiency by normalizing the precision $\left(\sigma_{\mathrm{T}}\right)$ by the square root of the imaging time $(N \times T R)$. The impact of $d \alpha$ errors was simulated by using the nominal $\alpha$ to estimate $T_{1}$ from biased data.

Phantom experiments were performed on solutions in seven vials ( $T_{1}=54$ to 1009 ms ), imaged on a clinical 1.5-Tesla MRI system (Signa EXCITE TwinSpeed, GE). A 3D fast SPGR sequence was employed to acquire 12 slices in $43 \mathrm{sec}(T E / T R=5.4 / 14 \mathrm{~ms}, \mathrm{BW}=15.6 \mathrm{kHz}, \mathrm{FOV}=20 \mathrm{~cm}$, matrix $=256 \times 192,3 \mathrm{~mm} \mathrm{slices}, 1 \mathrm{average})$. $T_{1}$ was estimated by repeating the sequence for different flip angles and fitting Eq.[2], assuming accurate knowledge of flip angles.

## RESULTS

Fig. 1 shows simulation results for $T_{1}=200-3000 \mathrm{~ms}$ in the absence of flip angle errors. Accurate measurements over the large $T_{1}$ range are possible using as few as two angles that are optimized for the specific $T R$ (Fig.1a). The pair $\left\{3,20^{\circ}\right\}$, optimized for $T_{1}=500 \mathrm{~ms}$ and $T R=5 \mathrm{~ms}$, is shown to illustrate errors incurred when improper angles are used. Also, efficiencies are comparable when appropriate angles are used and are generally higher with fewer angles (Fig.1b). Agreement between $T_{1}$ measured in phantoms and the true values is shown in Fig. 2 for the angle settings determined from simulations

The influence of flip angle errors is shown in Fig.3. The error introduced in $T_{1}$ estimates is proportional to the true $T_{1}$ value. Even an error of $15 \%$, not uncommon in many imaging coils such as the head coil, results in a $28 \%$ underestimate of $T_{1}$. Also evident in Fig. 3 is that the magnitude of error in $T_{1}$ cannot be reduced by choice of flip angles, since the relationship is independent of the number of angles or their values, as can be demonstrated from the analytic solution of Eq.[3].

## CONCLUSIONS

Simulation and in vitro studies demonstrate that rapid, accurate 3D $T_{1}$ mapping over a wide $T_{1}$ range is feasible using the VFA method. The flip angles must be optimized for the $T R$ and the $T_{1}$ range of interest, and, in general, efficiency is maximized when two angles are used. However, imprecise knowledge of the flip angle, due to B 1 field non-uniformity, can introduce significant error in the $T_{1}$ estimate that cannot be reduced by changing the number of flip angles or their values. Measurement of subject-specific B 1 variations is necessary to correct the measured $T_{1}$ through Eq.[3], especially at 3.0 T or higher. The time resolution of the study would not be affected if the B 1 map is acquired prior to $T_{1}$ measurement.

## REFERENCES

(1) Deoni SC, et al. MRM 2003; 49:515-526. (2) Wang HZ, et al. MRM 1987; 5:399-416. (3) Kay I, Henkelman RM. MRM 1991; 22:414-424.


Fig. 1 (a) Accuracy and (b) efficiency of $T_{1}$ measurements. Mean values $\mu_{T 1}$ and std.dev. $\sigma_{\mathrm{T} 1}$ normalized for imaging time are shown for select combinations of flip angles yielding accurate estimates over the range of interest. Values were obtained from 65,000 simulation runs.


Fig. $2 T_{1}$ measured in phantoms versus true values.


Fig. $3 T_{1}$ error due to error in flip angle $\alpha$. The ratio of the measured versus the true $T_{1}$ under simulated noisy conditions was averaged for all $T_{1}=200-$ 3000 ms . Flip angle combinations shown are identical to those in Fig. 1 (see legend).

