Using Longitudinal Magnetization Decay to Measure Long-Range ^{3}He Diffusion: Theoretical Analysis of Parameter Variations on Measurement Accuracy

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Introduction: The ^{3}He diffusion coefficient has typically been measured from the decay of transverse magnetization over a period of several milliseconds [1,2]. Recently, a new method was introduced to measure "long-range" diffusion over a period of seconds from the decay of "tagged" longitudinal magnetization [3]. Many different parameters may influence the accuracy of such long-range diffusion measurements. The purpose of this study was to use theoretical simulations to investigate the influence of B1-inhomogeneity, signal-to-noise ratio (SNR) and the number of images on the accuracy of long-range diffusion measurements.

Methods: Our model was based on a simulated axial image of the lung (Fig. 1). The spin density and T1 were assumed to be spatially uniform. Sinusoidal spatial modulation (tagging) of the longitudinal magnetization was simulated from the equations in reference 3. The effects of T1 decay, diffusion and the imaging RF pulses were applied to the tagged longitudinal magnetization, and Gaussian white noise was added into the resulting images. The final simulated image intensity is given by equation 1:

$$I = I_o \left|\cos^2\theta - \sin^2\theta \cos(2\pi x/\lambda)\exp(-4\pi^2 D_{\text{sec}} t/\lambda^2)\right| \exp(-t/T1)\sin(\alpha)\cos^{(N_{PE}/2)(2m-1)}(\alpha) + N(x,t) \qquad \textbf{(Equation 1)}$$

Parameters used in the simulation were: matrix size: 36(N_{PE})*128; tagging flip angle (θ): 20–60°; long-range diffusion coefficient (D_{sec}): 0.07 cm^2/s; FOV: 300*300mm; tagging wavelength (λ): 30mm; number of images (m): 4-8; time between image acquisitions: 1.0s; TR: 4.6ms; TE: 2.2ms; T1: 20ms; imaging flip angle (α): 5-7°. The amplitude of Gaussian white noise was varied to yield selected SNR values.

The average value, and the cosine and sine components of the image intensity were determined from the simulated series of images according to the calculation scheme described in reference 3. The fractional modulation FM at position x was determined from equation 2 [3]:

$$FM(x) = \sqrt{C^2(x) + S^2(x)}\Big/ a(x), \qquad \textbf{(Equation 2)}$$

where C(x) is the cosine component, S(x) is the sine component, and a(x) is the average. At the same time, the signal-to-noise ratio of each image was calculated. This procedure resulted in a new series of images that contained the FM values defined for each pixel. According to the analysis of reference 3, this FM signal decay is only sensitive to signal attenuation from diffusion, so the FM values were fitted to a mono-exponential decay to determine the long-range diffusion coefficient on a pixel-by-pixel basis. In this procedure, pixels outside the simulated lung and near the border of the lung were excluded.

B1-inhomogeneity of the RF coil results in variation of the flip angles of the tagging and imaging RF pulses [4], which may affect the accuracy of the calculated diffusion coefficients. In addition, it would be expected that the underlying image SNR and the number of images in the series would also affect the results. To investigate the effects of these different parameters, we varied one parameter value while keeping the other parameter values constant, and then compared the simulated diffusion values with the assumed diffusion coefficient of 0.07 cm^2/s. The mean and standard deviation of the simulated diffusion values were determined for each set of parameter values.

Results: Representative simulated images showing the decay of the tagged longitudinal magnetization, and the corresponding calculated diffusion coefficient map, are shown in Fig. 1. The standard deviation of the simulated diffusion values increased with decreasing signal-to-noise ratio, and was about 10% of the mean value when the SNR of the first image in the series was 50. For an actual diffusion coefficient of 0.07 cm^2/s, the simulated mean value was closest to the actual value for the maximum number of images (8). However, for substantially higher values of the actual diffusion coefficient (e.g., 0.17 cm^2/s) and a given initial SNR, the mean value was more accurately predicted by using a smaller number of images (4). Also, we found that the method was very sensitive to the tagging flip angles. In particular, when the tagging flip angle was greater than 45°, the simulated diffusion values were, as shown in Fig. 2, far different than the assumed value of 0.07 cm^2/s.

Conclusion: Using the longitudinal magnetization decay offers a way to measure the long-range diffusion displacement. However, the accuracy of this method is influenced greatly by several measurement parameters, particularly the tagging flip angle.

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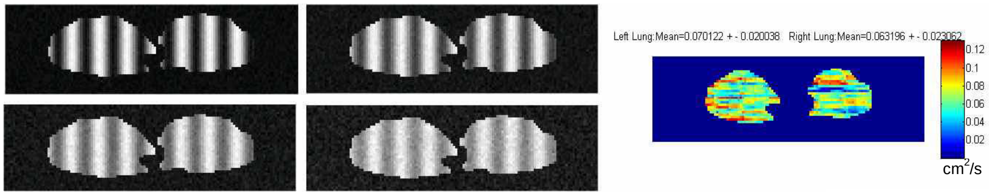

Fig. 1. Representative simulated images showing the decay of the tagged longitudinal magnetization (**left**), and the corresponding calculated diffusion coefficient map (**right**). The SNR for the first image (upper left) was approximately 60.

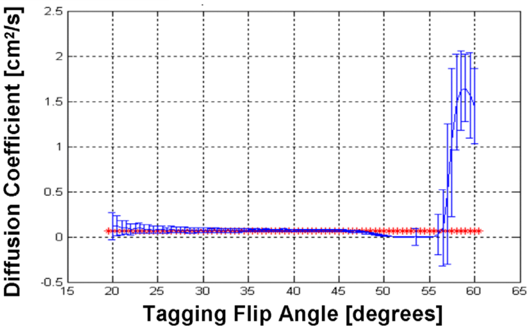

Fig. 2. Relationship between the simulated diffusion values and the tagging flip angle. The mean (solid blue line) and standard deviation (blue error bars) of the simulated diffusion coefficients were calculated from the whole lung. The red line indicates the assumed diffusion coefficient of 0.07 cm^2/s.