Application of Kalman Filtering to Detection and Estimation of Non-stationary fMRI Signals

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Introduction: Most fMRI time series analyses require the assumption that the underlying stochastic processes are *stationary*; i.e., that the statistics of the random processes do *not* vary over time. This assumption is not always valid. Changes over time in the statistics of the time series due to (1) abrupt subject motion, and/or (2) slow changes in the vascular hemodynamic response due to physiological or neuronal processes, will result in fMRI time series behaving in a non-stationary manner. A powerful tool for analysis of non-stationary time series is the Kalman filter [1]. Here, we describe an implementation of the Kalman filter for analysis of non-stationary fMRI time series.

Theory: Implementation of the Kalman filter includes a model of the system under study, as well as a model of the measurement process. For fMRI experiments, the system is the voxel [2]; specifically, the voxel hemodynamic response, which occurs in continuous time. The measurement process is discrete, occurring at intervals of 1 TR. Our "continuous-discrete time extended Kalman filter" [1] is based on the matrix equations:

System model: $$d\mathbf{x}/dt = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{G}(t)\mathbf{w}(t) + \mathbf{L}(t)\mathbf{u}(t)\,; \qquad (1)$$

Measurement model: $$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}(t_k)) + \mathbf{v}_k\,; \qquad (2)$$

where $\mathbf{x}$(t) = system (voxel) state vector, $\mathbf{x}(0) \sim N(\mathbf{x_0},\mathbf{P}(0))$; $\mathbf{w}$(t) = random forcing function, $\mathbf{w}(t) \sim N(\mathbf{0},\mathbf{Q}(t))$; $\mathbf{u}$(t) = deterministic input (stimulus function); $\mathbf{z_k}$ = (fMRI) measurement vector; and $\mathbf{v_k}$ = measurement noise, $\mathbf{v_k} \sim N(\mathbf{0},\mathbf{R_k})$. Note that this model requires an "extended" Kalman filter since $\mathbf{h}_k$ is a *nonlinear* function of the state variables. We modeled the system state using the 5 element vector $\mathbf{x}' = [\,b \quad \delta b \quad y \quad \delta y \quad g\,]$, where b = baseline, δb = baseline drift rate, y = hemodynamic response, δy = time rate of change of hemodynamic response, and g = gain factor relating fMRI measurement amplitude to the hemodynamic response. The hemodynamic response was modeled using a linear 2nd order differential equation with constant coefficients:

Hemodynamic response: $$d^2y/dt^2 + 2\zeta\omega_n\,dy/dt + \omega_n^2 y = \omega_n^2 f(t-\Delta t)\,, \qquad (3)$$

where $f(t)$ is the input, and ζ, ω_n, and Δt are constants which describe the shape of the hemodynamic response.

Materials and Methods: Non-stationary time series analysis was considered for two cases. In the first set of data, the subject was asked to intentionally make a small movement three or four times during a random event, bilateral finger-tapping experiment. For the second set where there was no apparent subject motion, changes over time in the process were attributed to variations in the vascular hemodynamic response of the system. BOLD weighted images were obtained using single-shot gradient-echo EPI readout. Both block design and multiple event-related experiments were performed.

Results and Discussion: The estimated motion parameters, for one particular run, are illustrated in Fig.1(a). These estimated motion parameters are used to perform motion correction. The time series for a single voxel, *after* motion correction, is shown in Fig.1(b). Note that the motion correction does *not* remove all of the signal distortion due to subject motion. Hence, the least squares fit (red line) of the random binary stimulus function, as illustrated in Fig.1(c), is rather poor. This is due to the fact that the least-squares-fit assumes that the underlying random process is stationary. However, using the Kalman filter, it is possible to allow for non-stationarity. Using the estimated motion parameters, the Kalman gain can be adjusted to, in effect, allow a resetting of the baseline when large subject motion occurs. The Kalman gain element (1,1) is displayed in Fig.1(d). The Kalman filter estimate (green line) is displayed in Fig.1(e).

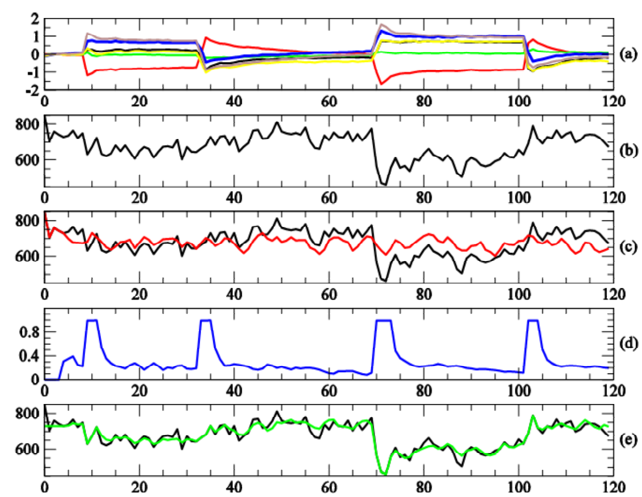

Figure 1: Kalman filter estimation of fMRI signal.

Here we clearly see the advantages of the Kalman filter. The Kalman filter estimate (green line) is a fit of the baseline plus the signal. Whereas the least-squares-fit treats all errors as measurement noise, the Kalman filter accounts for system disturbances that can have a long term impact on the fMRI signal resulting in permanent change in the baseline.

References

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