# Maximum *a posteriori* Independent Component Analysis (MAP-ICA): A Bayesian framework for incorporating prior information into ICA analyses of fMRI data

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**Introduction**

Data-driven approaches such as Independent Component Analysis (ICA) [1] for analysis of functional MRI (fMRI) data are an attractive alternative to hypothesis-driven methods such as the General Linear Model (GLM). A modified ICA technique is proposed which incorporates prior information about the expected hemodynamic response function (HRF) into a maximum *a posteriori* (MAP) ICA framework. The MAP-ICA procedure is modeled on the standard two-step PCA reduction/ICA decomposition procedure but includes prior information about the task regressors into the PCA and ICA algorithms. To incorporate the prior information, the probabilistic PCA (PPCA) [2] and Bayesian ICA [3] frameworks are used for the PCA and ICA steps, respectively.

**Theory**

Assuming *n* voxels in the brain, *m* acquired time frames, and *p* assumed components, the basic MAP-ICA model is $Y = A\,S + E$ where *Y* is an *m*-X-*n* data matrix, *A* is an *m*-X-*p* mixing matrix, *S* is an *p*-X-*n* matrix of independent sources, and *E* is an *m*-X-*n* matrix of residuals. The columns of *A* are the associated time courses for each ICA component. Prior information is assumed available for the first *q* ($q < p$) task-related regressors and will be incorporated in the first *q* columns of *P(A)*, where *P(A)* is the prior on the mixing matrix *A*, with the next *p* - *q* columns of *P(A)* being a uniform distribution (the ordering of ICA components being completely arbitrary). As in standard ICA, the first step consists of a PCA analysis. In order to incorporate the prior information on *A* we use a PPCA framework [2]. The EM-PCA algorithm [4] is modified by replacing the M-step with a gradient ascent on *A*, leading to the following iterative scheme:

$$S \rightarrow A^{+}Y, \sigma^2 \rightarrow E((Y-AS)(Y-AS)^T), A \rightarrow A + \mu(\frac{1}{\sigma^2}(YS^T - ASS^T) + \frac{P'(A)}{P(A)})$$ where $\mu$ is the learning rate parameter. The timecourses *A* found from the PPCA are used as the starting point for the gradient-ascent MAP-ICA algorithm. For computational ease the gradient ascent in MAP-ICA is performed on *A* rather than $W = A^{+}$. Using the previously derived [3,5] gradient update rule for *W*, identities from matrix calculus, and the prior information on *A*, the MAP-ICA update rule is derived as:

$$\Delta A = (I - AW)\Delta W_0{}^T (A^T A)^{-1} - W^T \Delta W_0 W^T + \mu \frac{P'(A)}{P(A)}$$ where $\Delta W_0 = \mu(A^T + \frac{P'(S)}{P(S)} Y^T), S = WY, P(S) \equiv$ prior source density.

**Materials and Methods**

Simulated data was generated via routines written in IDL (Research Systems Inc., Boulder, CO). A zero-mean, unity standard deviation Gaussian noise background of 10,000 voxels and 100 time points was generated., using a 1/*f*-like + white noise structure obtained empirically from the noise power spectrum of fMRI data obtained at 3T. Each voxel time course was normalized to have a standard deviation determined from a uniform distribution ranging from 0.5 to 1.0. Three sources were added with time courses consisting of sine waves of frequencies 0.017 Hz, 0.025 Hz, and 0.033 Hz, and amplitudes of 1 (peak-to-peak). "Active" voxels were randomly selected from one-sixth of the total number of voxels and were assigned intensities from a uniform distribution between 0.5 and 1.5. In addition, twenty super-Gaussian "confound" sources were generated by taking the square of a Gaussian distribution (keeping the sign) and normalizing to unity variance. The confound sources were added to the data using random time courses normalized to the same variance as the task regressors. The MAP-ICA procedure was performed both using fifteen components and fifty components. The starting point of the PPCA procedure was determined by corrupting the task regressors by additive Gaussian random noise such that the cross-correlation between the true and corrupted regressors was 0.75 and assigning them to the first three columns (the other columns were taken from a random Gaussian distribution). A uniform prior was used for all time courses having a correlation coefficient R > 0.75 with the noise-corrupted task regressors. The accuracy of MAP-ICA was measured by computing the cross-correlation coefficient of each source found via MAP-ICA to the reference source, and the cross-correlation coefficient of each regressor found via MAP-ICA to the true regressor. For comparison, the data was also processed using standard ICA. The data was reduced to 15 or 50 components via standard PCA, and the fastICA algorithm [6] used. Cross-correlation coefficients were computed between each source found from standard ICA and the reference source (using the source with the highest absolute cross-correlation coefficient), and between each regressor found via standard ICA and the true regressor. The data was also processed via a standard GLM, using the known regressors. The simulation was repeated 100 times and the results averaged over the 300 sources.

**Table 1.** Comparison of maximum *a posteriori* ICA (MAP-ICA) with standard ICA (sICA), using 15 and 50 components (NC) in the model. Results given are average correlations with the sources (SC) and associated time courses (TC) for simulated data with 3 task sources, 20 confound sources, and temporally autocorrelated noise (100 iterations X 3 sources = 300 data points). For reference the results are also given for a standard GLM analysis.

| Method | NC | SC | TC |
|---|---|---|---|
| MAP-ICA | 15 | 0.77 | 0.997 |
| sICA | 15 | 0.33 | 0.47 |
| MAP-ICA | 50 | 0.77 | 0.996 |
| sICA | 50 | 0.24 | 0.43 |
| GLM | 23 | 0.75 | 1.0 |

**Results and Discussion**

The MAP-ICA procedure significantly outperforms standard ICA (Table 1) in terms of accurately estimating the sources and associated time courses. MAP-ICA also displayed a slight improvement compared to the GLM for estimating the sources, since substantial temporal autocorrelation was present in the simulated data, and the GLM parameter estimates are only the best linear unbiased (BLUE) ones under the condition of white Gaussian noise. The use of prior information renders MAP-ICA more sensitive to weaker sources, and more robust to misspecification of the number of sources present in the data. The performance of standard ICA is sensitive to achieving the correct amount of data reduction [7], but finding the optimal amount of PCA reduction is problematic using typical methods (e.g. [8]) which assume a white noise background. MAP-ICA also appears to be somewhat robust to misspecification of the prior on *A*, since the prior used was not optimal (it was uniform for time courses with R > 0.75 with the noise-corrupted, not true, task regressors), although, as with all Bayesian analysis techniques, proper specification of the priors is critical for optimal performance. Furthermore, MAP-ICA readily allows for incorporation of prior information on the expected spatial location of cortical activation in the source prior *P(S)*. Thus MAP-ICA may provide an efficient method for Bayesian spatiotemporal analysis, typically highly computationally intensive with approaches such as Markov random fields [9].

**Conclusion**

A method for ICA analysis of fMRI is proposed which incorporates prior knowledge about the hemodynamic response into the Bayesian ICA formalism. The MAP-ICA technique is more robust to misspecification of the number of components present, and more sensitive to weaker sources, as compared to standard ICA. MAP-ICA may also provide a computationally efficient framework for Bayesian spatiotemporal analysis of fMRI data.

**References**

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