Transverse Relaxation in a Paramagnetic Microvascular Network as a Test of Locality of the Dipole Field

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Introduction

Diffusion in mesoscopic gradients of magnetic field induced by paramagnetic blood provides an essential contribution to the contrast in functional MRI, and the main contribution to signal variations obtained in dynamic perfusion imaging. These imaging modalities have a potential to evaluate morphology and functioning of cells and microvessels at a length scale much below the nominal imaging resolution (1). To realize this promising possibility in practice, one needs to develop quantitative theoretical models of the relation between the tissue structure and its effect on the transverse relaxation. In the present work we make a step in this direction, by demonstrating how one can reduce the complicated problem of calculating the susceptibility-induced T_2^* relaxation (which requires the knowledge of the autocorrelation function of the local Larmor frequency) to the much simpler problem of calculating the autocorrelation function of the spatially inhomogeneous susceptibility. In doing so we rely on the effective *locality* of the interaction between the nuclear spins and the paramagnetic matter (1,2).

This theoretically and numerically proven simplification can rationalize a well-known observation, that the simple chemical exchange model provides surprisingly good results for the transverse relaxation far away from its original applicability range.

Theory

We model the tissue by a suspension of identical objects, that are permeable, weakly paramagnetic, and arbitrarily placed and oriented. Assuming the dephasing due to the single object to be small, it can be shown (2) that, due to averaging over random orientations of the objects, the net effect of dephasing due to the magnetic field induced by the objects occurs inside them, i.e. the effective interaction between spins and objects is contact (*locality*). Moreover, this interaction results in a non-exponential transverse relaxation that strongly depends on the shape of the objects when their volume fraction is fixed (1).

We consider as an important example the spin dephasing in the diffusion-narrowing regime (DNR) by a vascular network, which is modeled as a set of randomly placed and oriented homogeneous cylinders. The vessel arrangement is illustrated in Fig. 1 for a simplified case of their parallel orientation (shown is a sample cross-section in the plane perpendicular to the vessels). The Larmor frequency offset induced by a vessel with magnetic susceptibility χ takes the form

$$\omega(\vec{r}) = \delta\omega \frac{R^2}{r^2}\cos 2\varphi \sin^2\vartheta \quad \text{for } r>R \text{ and } \omega(\vec{r}) = \delta\omega\left(\cos^2\theta - \frac{1}{3}\right) \quad \text{for } r<R \quad , \ \delta\omega = 2\pi\chi\gamma B_0 \qquad [1]$$

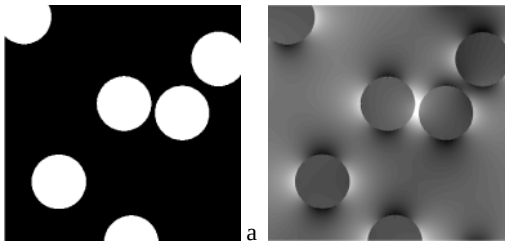
Here R is the vessel radius, φ is the polar angle in the plane orthogonal to the vessel and θ is the tilt angle of the vessel. The locality of the dipole field (2) implies that this field can be replaced with the following expression, which is much simpler:

$$\omega(\vec{r}) = 0 \quad \text{for } r>R \text{ and } \omega(\vec{r}) = \lambda\cdot\delta\omega \quad \text{for } r<R, \ \lambda = (11/72)^{1/2} \ . \qquad [2]$$

Fig. 1 illustrates the significant difference between expressions [1] and [2].

Results of Monte Carlo Simulations

We corroborate the above findings of the general theory (1,2) by Monte-Carlo simulations. The method of simulation is described in (3). Results are obtained for the free induction decay (measured with the gradient echo) and for the spin echo. Fig.2 represents a comparison between the simulation performed with the original field [1] and with the field replaced with equation [2]. The data are in a perfect agreement with the theory (1) which relies on the locality of the dipole field (2).



a b

Fig 1. Randomly positioned parallel cylinders (a) and the induced frequency shift in a horizontal magnetic field (b), shown is the sample cross-section.

Discussion

In the diffusion-narrowing regime, the locality of the dipole field, confirmed in the present numerical simulation, rationalizes the apparent success of the chemical exchange model (CEM). Indeed, the possibility to substitute the complex field pattern of Fig.1b by the simple indicator function of Fig.1a naturally captures the main feature of the CEM, which is the presence of two molecular pools, each with a definite shift of the Larmor frequency and a residence time. A superficial comparison between Figs. 1a and 1b shows how far the CEM is from the reality. The theory represented by eq. [2], provides the connection between the CEM (where the pools are understood as those inside and outside the vessels), and the dephasing in the true field of Fig.1b. One crucial difference is that the theory (1) treats the diffusion realistically, in contrast to CEM, in which its effect is approximated as hopping between the two pools.

To conclude, it is confirmed by Monte Carlo simulations that the dephasing in the diffusion-narrowing regime is governed by the mesoscopic susceptibility profile as it follows from the effective locality of the dipole field. This establishes the connection between tissue structure and transverse relaxation.

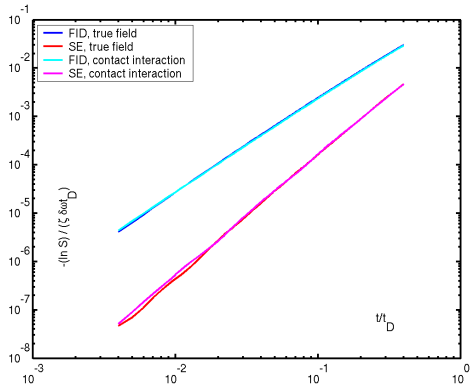


Fig. 2. Results of Monte Carlo simulations for a three-dimensional distribution of vessels with the field calculated according to eqs [1] and [2].

References

(1) VG Kiselev, DS Novikov. Phys Rev Lett. 89(2002)278101, Phys Rev Lett.91(2003)029802. (2) DS Novikov, VG Kiselev. abstract submitted to this Meeting. (3) BF Jensen et al. proc. ESMRMB 2004, Copenhagen, abstract 269.