The Choice of Interpolation Scheme for Resolving Crossing Fibres in Diffusion Imaging Data

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Introduction

Diffusion tensor imaging has proved an extremely powerful technique for imaging white matter structure *in vivo*. Recently, attention has turned to the issue of resolving multiple fibre directions within a single image voxel in order to probe regions in which fibres cross. These structures cannot be resolved by tensor imaging and require more involved analysis of higher angular resolution data. One proposed technique for resolving fibre-crossing structure is q-ball imaging [1] in which a Funk-Radon transform is applied to the data in order to obtain an Orientational Distribution Function (ODF). This transform is performed by integration around contours describing an "equator" defined by each gradient direction.

The question of interpolation on a sphere is central to q-ball analysis. Most importantly, in performing the Funk-Radon transform, each contour integral requires that the data in question be interpolated to a series of points along its length. Clearly the accuracy of the interpolation method used is of crucial importance. Additionally, since the fibre ODF resulting from the application of q-ball is purely numeric (as opposed to, say, a vector of spherical harmonic or expansion coefficients) and that any tractography algorithm based upon its results will require to find directional maxima in the ODF, further interpolation between transformed data points may be required in order to extract underlying fibre directions.

Although not widely recognised, interpolation on a sphere is an essentially different problem to that in a plane. The question of evenly distributing an arbitrary number of points on a sphere is still an unsolved problem --no analytical expression for exactly tessellating the sphere evenly for n points is known to hold for all n-- and optimisation techniques (such as minimum energy placement), whilst good at finding local minima on their potential landscapes have difficulty in locating the global minimum that would be free of fault-like discontinuities between locally regular regions.

Methods

We consider three forms of interpolation: (i) Spherical radial basis functions (SRB) [2], which are a popular choice, and work essentially by fixing a weight to each data point as a suitable function of its distance from the interpolation point. (ii) A simple least-squares fit of spherical harmonic coefficients. This is performed using the method of Alexander et al [3]. (iii) Hyperinterpolation, which is a less well-known technique involving a projection of the data into an orthogonal polynomial space [4].

Each algorithm was passed data points evaluated in a number of gradient directions assigned by minimum energy placement from a known function and a controlled level of additive noise. Each algorithm was then used to interpolate values at a large number of points on the sphere (approx 100000) and the result compared with the exact value from the known function. Results for mean error were extracted. As a geometry pertinent to fibre-crossing applications, the known function was chosen to be the real part of a second-order spherical harmonic, Re{$Y_{2,2}$} with noiseless data as well as SNRs of 10, 5, 3 and 2. Identical data were passed to each technique.

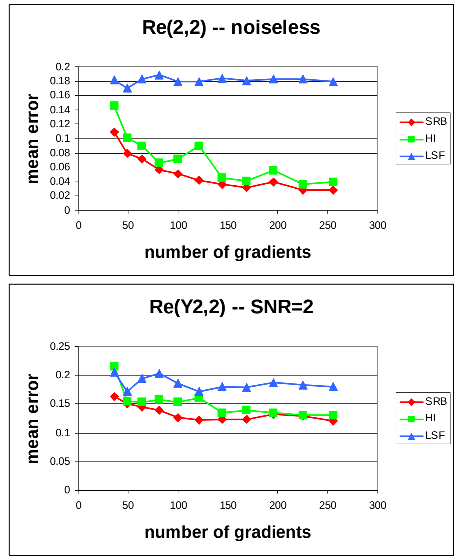

Fig-1: Sample results from mean interpolation error. In the noiseless cases SRB and Hyperinterpolation perform well. Noise induces a greater effect on these two techniques than least squares fitting.

Results

Shown in Figs 1 and 2 are the results for the second order harmonic. Although all three techniques are extremely accurate with noiseless data, the presence of noise causes a significant effect. Results for SRB and Hyperinterpolation show an increase in mean error by an order of magnitude at low SNR, whereas least-squares shows an increase in the same measure on the order of 10% (see Fig-1). Noise causes both SRB and Hyperinterpolation to exhibit distortion in the interpolated function; this distortion manifests itself differently in least-squares fitting, with the function consistently over-estimated in one "fibre" direction and underestimated in the other (see Fig-2). Least-squares results are also subject to difficulties with fitting more amorphous underlying functions.

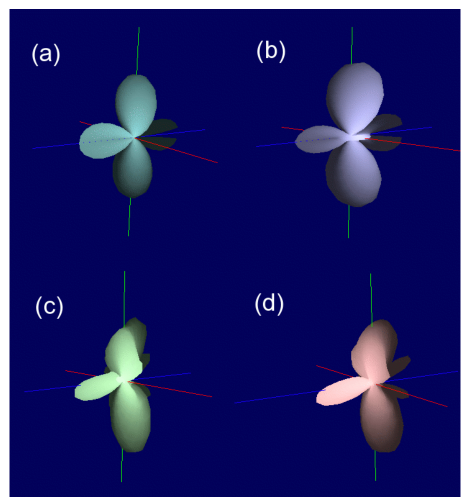

Fig-2: Second order harmonic surfaces in the presence of noise. (a) The exact underlying function, (b) least-squares fit, (c) SRB interpolation and (d) Hyperinterpolation. Whilst (b) preserves orientational maxima, (c) and (d) have a lower mean error and do not favour one pair of lobes over another to the same degree.

Discussion

The choice of interpolation technique is important in making the most of interpolatory accuracy and, by extension, any other techniques making use of interpolation such as q-ball. Least-squares fitting performs well in some cases, but comes with its own set of complications such as producing singular matrices for inversion during fitting.

The effect of noise is significant when choosing an algorithm and it is important to be aware of an algorithm's robustness to it. Both SRB and Hyperinterpolation exhibit distortion in the placement of directional maxima to a similar degree, greater than that seen for the least-squares fit, but overall mean error is reduced.

Choosing an algorithm is a trade-off between different constraints and on balance the benefits of SRB and Hyperinterpolation in reduced mean error would seem to recommend them over least-squares. Of the two SRB performs better (see Fig-1). However if computation time is a limiting factor, Hyperinterpolation is a significantly faster algorithm.

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References

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