# Geometric Extraction of Two Crossing Tracts in DWI 

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## INTRODUCTION

Fitting diffusion tensor data from heterogeneous white matter voxels to a single tensor can lead to errors in computed tract direction. High angular resolution diffusion-weighted imaging can be used to increase the directional information for WM tractography in order to try to address this problem. In Ref. [1] 126 directions were acquired with 8 averages; and in Ref. [2] 162 directions with 3 averages were used. The duration of these scans is in most cases too long for clinical applications and fitting the data to a general two-tensor model is difficult. Here, we demonstrate that whole brain DWI data acquired in 6 minutes is sufficient for effectively resolving two tract directions when only two tracts are assumed to populate the voxel. From a geometrical viewpoint, the two tract assumption leads to the following conclusion: Both tracts are in the plane defined by the first two principal axes of the single tensor fit, i.e., the eigenvector corresponding to the smallest eigenvalue defines the normal vector to this plane. An additional constraint used in this preliminary analysis is that the apparent diffusion coefficients parallel and perpendicular to the tracts, respectively, are the same for both tracts. This leads to the conclusion that the apparent diffusion constants of both tracts in the restricted directions are equal to the smallest eigenvalue calculated from the single-tensor model. The resulting "two crossing fiber" model has only 5 free parameters under the physically reasonable assumptions that the two tracts within the voxel both exhibit cylindrical symmetry and that there is no significant bending of the tracts within the voxel.

## METHOD

1. Diffusion-weighted dual-spin echo EPI data with 31 gradient directions and 31 slices was acquired at b-value $750 \mathrm{~s} / \mathrm{mm}^{2}$ on a GE 3 T .

TE/TR/FOV/matrix/slice $=86.9 \mathrm{~ms} / 11.5 \mathrm{sec} / 24 \mathrm{~cm} / 128 \times 128 / 3.5 \mathrm{~mm}$. Total imaging time was 6 minutes ( 1 average).
2. The single tensor that best described the diffusion in each voxel was calculated.
3. The first two eigenvectors $\hat{e}_{1}, \hat{e}_{2}$ of the single tensor define the plane of the tracts, and the third eigenvector, $\hat{e}_{3}$ (corresponding to the smallest eigenvalue $\lambda_{3}$ ) defines the perpendicular to this plane. Therefore, we proceeded by transforming the gradient vectors, $\mathbf{G}$, into the orthonormal coordinate system defined by $\left[\hat{e}_{1}, \hat{e}_{2}, \hat{e}_{3}\right]$. In this coordinate system both tensors are of the form: $\left(\begin{array}{ccc}D_{1} & D_{3} & 0 \\ D_{3} & D_{2} & 0 \\ 0 & 0 & \lambda_{3}\end{array}\right)$, and
this now becomes a 2D problem.
$D_{3}=\sqrt{\left(\lambda_{3}-D_{2}\right)\left(\lambda_{3}-D_{1}\right)}$ due to cylindrical symmetry.
4. Out of the acquired gradient directions, those that have large components in direction $\hat{e}_{3}$ (perpendicular to the plane of the 2 tracts) contribute very little to the discrimination between tract directions and thus just add noise to the calculation. Here, approximately $75 \%$ of the directions were kept.
5. The signal attenuation equation was solved:
$S=f_{\alpha} \exp \left(-b \mathbf{G} \mathbf{D}_{\alpha} \mathbf{G}^{T}\right)+f_{\beta} \exp \left(-b \mathbf{G} \mathbf{D}_{\beta} \mathbf{G}^{T}\right)$, where
$b=\gamma^{2} \delta^{2}(\Delta-\delta / 3), f_{\alpha}$ and $f_{\beta}$ are the relative amplitudes of the signal from each tract, and the gradients and diffusion tensor are given in the new coordinate system. We used a non-linear least-squares method implemented by Mathworks in Matlab.

## RESULTS

Figure 1 shows an example of the results of this calculation in an area of the brain where Westin's planar anisotropy index, $C_{p}$ [3], is high (white patches). The data were analyzed on a pixel-by-pixel basis without taking into account neighboring pixels, thus groups of pixels showing continuity of crossing tracts support the validity of the analysis. At obvious interfaces between tracts, the crossing tract analysis finds both bordering tract directions. Both the angular separation between the two calculated tracts, and fractional size of the $2^{\text {nd }}$ component


Figure 1. Calculated tract directions ( $1^{\text {st }}$ eigenvector) from a region of interest in the brain. Regions where the planar anisotropy coefficient is high are shown in white. Vectors with a through-plane component are indicated by red circles. Top Right: Single tensor analysis.
Bottom: Two directions calculated using the 2 crossing fiber model. correlate positively with $C_{p}$, (data not shown) giving added credence to this method.

## DISCUSSION

We achieve robust extraction of 2 directions from whole brain DWI data acquired in 6 minutes. Calculation of the tract directions was achieved by taking into account the geometry associated with two separable fiber bundles within each of the analyzed voxels and by utilizing the single-tensor information. The underlying constrained two-tensor model has only 5 free parameters to fit, compared to 14 parameters in a general two tensor model. Future research directions include the extension to local neighborhood analysis and utilization of the results in tractography.

## REFERENCES

[1] D. Tuch et al, MRM 48:577 (2002). [2] A.L. Alexander et al, MRM 45:770 (2001). [3] C-F. Westin et al, Proc. ISMRM 1742 (1997).

