Effects of Finite-duration Gradients in q-Space MRI of Restricted Diffusion

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Introduction: The characteristic behavior of restricted diffusion in q-space MRI is potentially very useful since it offers the possibility of probing the micro-structure (1). Therefore q-space imaging has found its way to biomedical MRI, not only on animal systems but even on clinical systems (2). An important problem on clinical scanners is that the theoretical conditions for q-space imaging can not be fulfilled: impulsive strong gradients can not be used today on patients. As a result, the effects of the finite duration of the diffusion encoding gradients become very important when interpreting the data.

Different approaches were used to include these effects on the MR-signal: the Gaussian Phase Approximation (GPA) (3), the Short Gradient Pulse (SGP) approximation (1), the partial GPA applied to the SGP-case (4) and a general matrix method based on the SGP case (5). We have generalized the partial GPA to finite duration gradients and compared the different approaches for one dimensional restricted diffusion between planar boundaries. We have also investigated the effect of choosing the diffusion time on the reconstructed probability displacement function.

Methods: For restricted diffusion between reflecting planar boundaries at $x = 0$ and $x = a$, the propagator can be written in terms of an eigenmode expansion:

$$p(z_0, z, t) = \frac{1}{a}\left[1 + 2\sum_{n=1}^{\infty} \cos\left(\frac{\pi n z_0}{a}\right)\cos\left(\frac{\pi n z}{a}\right)\exp\left(-\frac{\pi^2 n^2 D t}{a^2}\right)\right] \qquad [1]$$

with z_0 the initial position, z the position at time t of the diffusing spins and D the diffusion coefficient. In the SGP approximation, the MR signal becomes the Fourier transform of the averaged propagator:

$$S(q) = \int P(Z,\Delta)\exp(i2\pi qZ)dZ \qquad \text{with} \qquad P(Z,t) = \int p(z_0, z_0 + Z, t)dz_0 \qquad [2]$$

Here, the q-space coordinate is given by $q = \gamma g \delta$ with g the amplitude of the diffusion encoding gradients, δ the duration of one lobe and Δ the time separation between the onsets of the two lobes in the spin echo sequence. By decomposing a finite duration gradient waveform into a sequence of impulses and using multiple propagators, the MR signal can be expressed into matrix form. In the GPA, the MR signal decays exponentially:

$$S(q) = \exp(-\beta) \qquad \text{with} \qquad \beta(q) = \frac{\gamma^2}{2}\int dt_1 \int dt_2\, g(t_1) < z(t_1)z(t_2) > g(t_2) \qquad [3]$$

and $< z_1 z_2 >$ represents the autocorrelation (second cumulant) which is obtained by using the propagator and performing an integration over both z_0 and z coordinates. In the partial GPA, partial cumulants are calculated by performing an integration only over the z coordinate so that the first cumulant does not vanish. In this case, the MR signal becomes:

$$S(q) = \int \exp\left(i\phi(q, z_0) - \beta(q, z_0)\right)dz_0 \qquad \text{with} \qquad \phi(q, z_0) = \gamma \int g(t) < z(t) > dt \qquad [4]$$

and $\beta(q, z_0)$ given by eq.[3b] but with the understanding that partial cumulants are used.

The MR signals obtained with these methods were compared whereby we considered the matrix method as the most correct method. In practice, q-space data is analyzed by taking a Fourier transform to obtain a probability displacement function which is interpreted as the averaged propagator. To simulate the effect of the choice of the diffusion time, we considered the averaged propagator at different proposed diffusion times in the literature and compared its Fourier transform with the MR signal (from the matrix method).

Results and conclusions: Figure 1 shows the MR signal versus $\theta = qa$ as obtained with the different methods for $\mu = D\Delta/a^2 = 0.6$ and $\varepsilon = \delta/\Delta = 1/8$ *or* 0. It can be seen that (as already known) the GPA is only valid for very small θ. The partial GPA gives the correct locations of the minima if $\varepsilon = 0$, but gives wrong signal values at other θ-values; for $\varepsilon \neq 0$ wrong results are obtained for all θ-values. Figure 2 shows that changing the diffusion time from $t_{diff} = \Delta$ to $\Delta - \delta/3$ or $\Delta + \delta$ does not change significantly the averaged propagator. As a result, the Fourier transforms do vary significantly (see Fig.3). Moreover, the resulting signals are completely wrong when compared to the matrix method ($\mu = 0.6, \varepsilon = 0.5$). Also, the correction scheme proposed in Ref (6) does not work.

In conclusion, when using finite duration gradients, the matrix method should be used to calculate the MR signal and the Fourier transform of it does not lead to the averaged propagator at a certain diffusion time in case of restricted diffusion.

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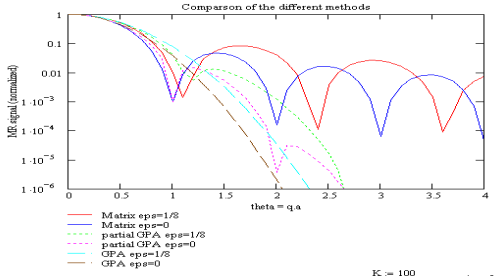

Fig.1: Comparison of the different methods.

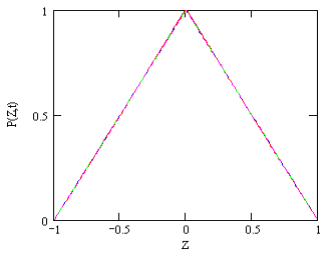

Fig.2: Effect of diffusion time on averaged propagator.

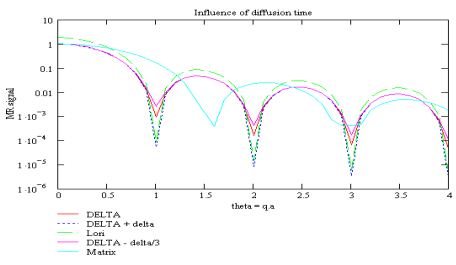

Fig.3: Effect of diffusion time on MR signal.