Anisotropy-optimized Lattice Index (ALI): a new formula for Diffusion Anisotropy Index

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Introduction

The Lattice Index (LI) is an established diffusion anisotropy index that quantizes the cohesiveness of the microstructure of the living tissues [1]. Originally introduced by Pierpaoli and Basser [2], LI is based on the sum of the dot products of 3 eigenvalue/eigenvector sets (obtained from diagonalization of 3x3 2^{nd} order Diffusion Tensor Matrix (D) between reference voxel and neighboring voxels). As a result, LI is rotationally invariant and less sensitive to the noise [1]. We believe a new formula of the anisotropic part of D may optimize the anisotropy detection while taking the full advantage of the mathematical properties in the original LI. The goal of the current study is to investigate this new formula for measuring diffusion anisotropy in tissues.

Theory

The symmetric 3x3 D matrix may be simplified to $D = \lambda_1 \hat{e}_1 \hat{e}_1^T + \lambda_2 \hat{e}_2 \hat{e}_2^T + \lambda_3 \hat{e}_3 \hat{e}_3^T$, where λ_i and $\hat{e}_i$ are i-th eigenvalue and eigenvector, respectively. According to Pierpaoli et al. [2], the "basic element" of LI between a reference voxel (R) and j-th neighboring voxel (N) is defined as $A_{dd}{}^j = (D_{a,R} \bullet D_{a,N}) / (D_R \bullet D_N)$. Denominator of the "basic element" is the dot product of D matrices (called tensor dot product), and the numerator is the anisotropic part of the tensor dot product $D_R \bullet D_N$. With the original LI, ***anisotropic*** part $D_{a,R} \bullet D_{a,N}$ is the tensor dot product $D_R \bullet D_N$ subtracted by the ***isotropic*** part (i.e. 1/3Trace(D_R)Trace(D_N)):

$$D_{a,R} \bullet D_{a,N} = D_R \bullet D_N - \tfrac{1}{3} Trace(D_R) Trace(D_N) = \sum_{u=1}^{3} \sum_{m=1}^{3} \lambda_{u,R} \lambda_{m,N} \left[(\hat{e}_{u,R} \bullet \hat{e}_{m,N})^2 - \tfrac{1}{3} \right] \quad \text{[eq.1].}$$

We would like to propose a new formula of the ***anisotropic*** part $D_{a,R} \bullet D_{a,N}$. Assuming that D has a combination of 3 basic geometric Tensor states (linear ($\lambda_1 >> \lambda_2, \lambda_3$), planar ($\lambda_1 \approx \lambda_2 >> \lambda_3$), and spherical ($\lambda_1 \approx \lambda_2 \approx \lambda_3$)), D was rearranged to have 3 terms, one term for representing each case:

$$D = (\lambda_1 - \lambda_2) E_L + (\lambda_2 - \lambda_3) E_p + \lambda_3 E_s \quad \text{[eq.2],}$$

where $E_L = \hat{e}_1 \hat{e}_1^T$ (linear), $E_p = \hat{e}_1 \hat{e}_1^T + \hat{e}_2 \hat{e}_2^T$ (planar), and $E_s = \hat{e}_1 \hat{e}_1^T + \hat{e}_2 \hat{e}_2^T + \hat{e}_3 \hat{e}_3^T$ (spherical) represent the Tensor basis of any Tensor. The anisotropic part of D (D_a) is identified as the first 2 terms of [eq.2], and further rearrangement results in following:

$$D_a = (\lambda_1 - \lambda_2) E_L + (\lambda_2 - \lambda_3) E_p = (\lambda_1 - \lambda_3) \hat{e}_1 \hat{e}_1^T + (\lambda_2 - \lambda_3) \hat{e}_2 \hat{e}_2^T = D_{a'1} + D_{a'2} \quad \text{[eq.3],}$$

where $D_{a'1}$ and $D_{a'2}$ are the resulting 2 terms from D_a rearrangement. Numerator of the "basic element" $A_{dd}{}^j$ is the dot product of [eq.3]:

$$D_{a,R} \bullet D_{a,N} = \sum_{k=1}^{2} \sum_{v=1}^{2} D_{a'k,R} \bullet D_{a'v,N} \quad \text{[eq.4].}$$

The final LI measures are found by the sum of the distance-weighted "basic elements" $A_{dd}{}^j$ divided by the sum of distance factors. The distance factors are defined as 1/(distance from the reference to neighboring voxel), with distance factor=1 for voxels that share the sides with reference voxel and $1/\sqrt{2}$ for diagonal voxels. The LI calculated with [eq.3] and [eq.4] has increased range of the final value (from [0 2/3] with original LI, to [0 1] with the new formula) and is optimized for more sensitive anisotropy detection. This new LI is thus referred to as ***Anisotropy-optimized Lattice Index (ALI)***.

Materials and Methods

Brains of 8 healthy subjects (4 males and 4 females), matched for demographic characteristics, were scanned with 1.5T GE Signa LX at the UC Davis Imaging Research Center (Sacramento, CA). Imaging parameters are following: DWI sequence with b=0/900, TE/TR= 90.4/8000, (2 images of b0, and 4 images per 6 gradient directions) for each of 19 slices (total image# = 494), 5 skips 0, FOV=240mm, 128x128 matrix. The DTI data were then processed offline to calculate the LI measures. Note that final LI measures in all cases have been multiplied by 1000 to give a better contrast. For each subject, Mean and Standard Deviation (SD) of LI measures are obtained in the ROI (Figure 1(a)) at anterior callosal white matter adjacent to Anterior Cingulate Cortex (ACC) on the oblique AC/PC slice. Specifically for this study, the "Signal-to-Noise Ratio (SNR, in dB)" is defined as 20*log of the Mean/SD ratio. Given the same D matrix and the same ROI, the mean SNR among 8 subjects is compared for LI vs.ALI.

Results and Discussion

The comparison of SNR between LI and ALI shows that ALI has increased SNR by average of 22.4% over original LI. The figures of LI and ALI show the difference in the signal strength and interpretation of LI values in the brain (Fig.1(b),(c)).

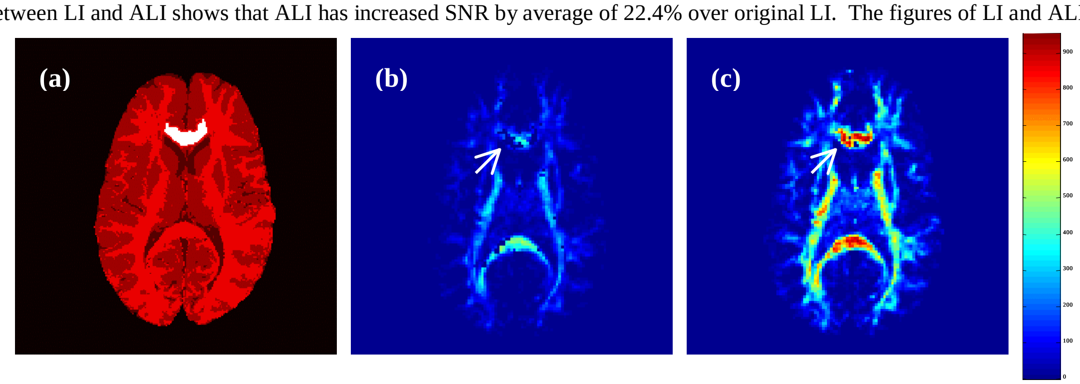

Figure 1: The representative images of (a) ROI on segmented brain, (b) Original LI, and (c) ALI. (b) and (c) are rescaled to 0-1000. (c) ALI shows overall higher intensity level than (b) LI, and visible interpretation differences are shown (e.g. at Frontal Corpus Callosum, shown in white arrow, low LI values vs. high ALI values).

References

1. Skare et al, MR Im., 2000.
2. Pierpaoli et al, MRM, 1996.

Table 1:SNR table of LI and ALI

	Mean	SD	SNR (dB)
LI	271.3	119.5	**7.14**
ALI	504.9	184.7	**8.75**