Effect of diffusion-encoding gradient duration on the displacement functions calculated in q-space and q-ball imaging

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Introduction: White Matter (WM) tracking is a diffusion MRI technique with great potential for studying WM structure in healthy subjects and patients. Several WM pathologies, e.g. multiple sclerosis, affect the diffusion properties of water in WM structures by changing the membrane permeability, and/or changing the size of the cells constraining the movement of water. Normal brain development also leads to changes in WM structure. It is thus important to understand the relation between the diffusion MRI signal and the structure of WM. Two MRI techniques that have shown promise in revealing WM structure are q-space imaging (QSI) *(1,2)* and q-ball imaging (QBI) *(3,4)*. The problem of using these techniques to study WM structure in living humans is that the structural interpretation of both QSI and QBI requires a simple relation between the diffusion MRI signal and the water displacement properties. Here, a simple relation means that the average dephasing for a fixed displacement must be analytically known, and must be independent of the local value of the diffusion coefficient *(5,6)*. If the duration of the diffusion encoding gradients, δ, is very small, then the relation between the diffusion MRI signal and the water displacement probability is simple. The problem in both QSI and QBI is that MRI in living humans needs a long δ. Other simulation studies *(5,6)* have shown that it is possible to obtain a simple relation between the diffusion MRI signal and the water displacement properties in many situations, even for a long δ. Those studies *(5,6)* were limited in that they did not use a realistic model of WM tissue, and only considered QSI and not QBI. Both problems are addressed in this abstract.

Theory: *QSI formalism:* In the most general case, the relation between physical displacement probability $P(\mathbf{r},\Delta+\delta)$ and MR signal intensity $E(\mathbf{g},\Delta,\delta)$, with **g** the diffusion-encoding gradient, is expressed by the integral $E(\mathbf{g},\Delta,\delta)=\int\cos(\eta_{true}\gamma\delta\mathbf{g}\cdot\mathbf{r})P(\mathbf{r},\Delta+\delta)\,d\mathbf{r}$, where $\eta_{\text{true}}\equiv\cos^{-1}\left(\mathrm{Re}\langle\mathbf{r},\Delta+\delta|e^{i\phi}\rangle_{\mathbf{g},\Delta,\delta}/[\gamma\delta\mathbf{g}\cdot\mathbf{r}]\right)$, and $\langle\mathbf{r},\Delta+\delta|e^{i\phi}\rangle_{\mathbf{g},\Delta,\delta}$ is the average dephasing for a fixed displacement **r** during the diffusion time Δ+δ *(5,6)*. In the case of free-diffusion, it was obtained *(5,6)* that η_{true} is independent of the medium's diffusion coefficient. In an actual QSI experiment, one attempts to invert the above integral using an η_{guess} that might be different from the correct η_{true} for that experiment. If η_{true} is independent of **g** and **r**, then the relation between the physical displacement probability $P(\mathbf{r},\Delta+\delta)$ and the experimentally obtained displacement probability $P_{MR}(\mathbf{r},\Delta+\delta)_{\eta_{guess}}$ is: $P_{MR}(\mathbf{r},\Delta+\delta)_{\eta_{guess}}=[\eta_{guess}/\eta_{true}]P([\eta_{guess}/\eta_{true}]\mathbf{r},\Delta+\delta)$. *QBI formalism:* If one is only interested in the orientation probability, instead of the displacement probability, then instead of QSI (1,2) it might be best to use QBI *(3,4)*. In the QBI formalism, the orientation distribution function (ODF) *(3,4)* in the direction of the unit vector **u** is $\Psi(\mathbf{u})\equiv\int_0^\infty P(b\mathbf{u},\Delta+\delta)\,db$. To obtain the ODF using MRI at a fixed q-amplitude *q'*, one uses $\Psi_{q'}(\mathbf{u})_{MR}\equiv\int E(\{q,\vartheta,\zeta\},\Delta,\delta)\,\delta(q-q')\,\delta(\zeta)\,q\,dq\,d\vartheta\,d\zeta$, where the vector **q** is now represented in cylindrical coordinates, with the **z**-axis pointing along **u**. In this coordinate system **q** is represented by amplitude q, angle ϑ and z component ζ. The relation between the ODF and the probability of displacement perpendicular to **u**, $P(\{r_{\mathbf{u}},\theta_{\mathbf{u}}\},\Delta+\delta)$, is obtained in (3,4). In our slightly altered version of QBI, we take into account the effects caused by the non-infinitesimal value of δ, and obtain $\Psi_{q'}(\mathbf{u})_{MR}=q'\int P(\{r_{\mathbf{u}},\theta_{\mathbf{u}}\},\Delta+\delta)\,J_0(\eta_{true}q'r_{\mathbf{u}})\,r_{\mathbf{u}}\,dr_{\mathbf{u}}d\theta_{\mathbf{u}}$ where J_0 is the zero-order Bessel function, which is similar to the corresponding equation in (3,4) and approaches the ODF $\Psi(\mathbf{u})$ for large $\eta_{true}q'$. *Limits of QSI and QBI formalisms:* For completely restricted diffusion inside a cylinder, the expression for η_{true}, if $[D_{in}\Delta]/a^2$ is small, is: $\eta_{\text{true}}\approx\sqrt{[\Delta-\frac{\delta}{3}]/[\Delta+\delta]}\sqrt{1-1.42\,[D_{in}\Delta/a^2]\,[\Delta/[\Delta-\frac{\delta}{3}]]}$. Three conclusions are drawn from the previous expression. The first conclusion is that, in the large *a* limit, $\eta_{\text{true}}=\sqrt{[\Delta-\frac{\delta}{3}]/[\Delta+\delta]}$. The second is that η_{true} is dependent on both **g** and **r** in the case of small *a*, which makes it impossible to obtain reliable water displacement probability information from $E(\mathbf{g},\Delta,\delta)$, be it either $P(\mathbf{r},\Delta+\delta)$ or $\Psi(\mathbf{u})$. The third is that the effect of restriction on η_{true} is a decrease from $\eta_{\text{true}}=\sqrt{[\Delta-\frac{\delta}{3}]/[\Delta+\delta]}$, in agreement with the results in *(5,6)*.

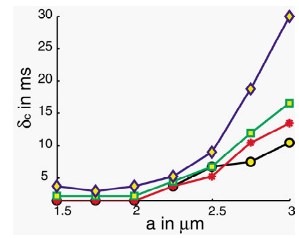

Figure 1: Plot of δ_c vs. cylinder radius *a*.

Simulation Results: We did computer simulations of water diffusion in WM-like tissue modeled as an array of permeable cylinders. For each value of δ, 3000 random walks are calculated. Each random walk has steps defined by a time-jump *dt* of 0.04 ms *(7)*, which corresponds to a step of amplitude $\sqrt{6D\,dt}$, where *D* is the diffusion coefficient of the medium at the step start. The values of the diffusion coefficient are 1.0 micron^2/ms if the step start is inside the cylinder, D_{in} *(8)*, and 2.5 micron^2/ms if the step start is outside, D_{ex} *(8)*. For all simulations, $\Delta+\delta$=60.0 ms, the maximum value of δ is $\delta=\Delta$, and |**g**| is 18.0 mT/m. The intra-extra exchange time $T_{in\text{-}ex}$ is 600 ms *(8,9)*, which for a cylinder of radius *a* corresponds to an intra-extra exchange probability $P_{in-ex}=\left[2a\sqrt{dt/[6D_{in}]}\right]/[T_{in-ex}-a^2/[8D_{in}]]$ *(7)*, and an extra-intra exchange probability of $P_{ex-in}=\sqrt{D_{in}/D_{ex}}P_{in-ex}$ *(7)*. If η_{true} obtained in the simulations stops being within 10% of $\eta_{\text{true}}=\sqrt{[\Delta-\frac{\delta}{3}]/[\Delta+\delta]}$ as we increase δ, we consider to have reached the highest acceptable δ value, δ_c. The value of δ_c for different cylinder radii *a* and angles between the cylinder axis and **g** are in Fig. 1. If $\delta>\delta_c$, then it is not possible to relate the diffusion MRI signal and the displacement probability in either QSI *(1,2)* or QBI *(3,4)*. In Fig. 1, different lines correspond to different angles between **g** and the axis of the cylinder: 90°-black circles, 88°-red stars, 86°-green squares, and 84°-blue diamonds.

Conclusion: QSI and QBI depend on δ differently, but both can determine WM structure accurately in typical experimental circumstances.

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References: (1) Y. Assaf, et al. *Magn Reson Med* **47**:115-26 (2002); (2) C.P. Lin, et al. *Neuroimage,* **19**:482-95 (2003); (3) D.S. Tuch, et al. *Neuron* **40**:885-95 (2003); (4) D.S. Tuch. *Magn Reson Med* **52**:1358-72 (2004) [In Press]; (5) N.F. Lori, T.E. Conturo, D. Le Bihan. *Journal of Magn Reson* **165**:185-95 (2003); (6) N.F. Lori, T.E. Conturo. p. 1202, ISMRM 2004, Kyoto; (7) C. Meier, W. Dreher, D. Leibfritz. *Magn Reson Med* **50**:500-9 (2003); (8) C.-L. Chin, et al. *Magn. Reson. Med.* **47**:455-60 (2002); (9) C. Meier, W. Dreher, D. Leibfritz. *Magn Reson Med* **50**:510-4 (2003).