Is the "Bi-Exponential Diffusion" Bi-Exponential?

V. G. Kiselev[1], K. A. Il'yasov[1]

[1]Medical Physics, Department of Diagnostic Radiology, University Hospital Freiburg, Freiburg, Germany

Introduction. Diffusion in the human brain shows a nonlinear dependence between the logarithm of the signal attenuation due to the diffusion weighting, ln *s*, and the b-factor, *b* (1-3). This is often interpreted as an indication of the presence of two compartments with different values of the diffusion coefficient, *D*. However, any attempt to identify these compartments morphologically has failed. An alternative interpretation of experimental data assumes that it the restricted nature of diffusion in tissues that results in the bi-exponential behaviour. This is supported by several theoretical examples (4,5). In this work, we revive a very basic theoretical reason for a polynomial form of ln *s*(*b*) and discuss its relation with the bi-exponential description.

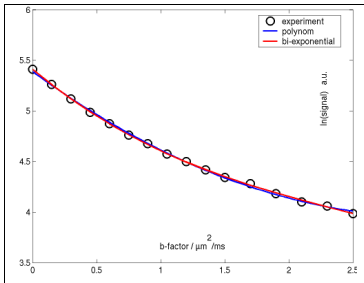

Fig.1. Data obtained in white matter

Theory. The signal attenuation caused by the diffusion weighting is the average value of phase factors of individual spins. As such, it has a representation as a cumulant expansion, which is a Taylor expansion in the even powers of the applied gradient, *g* (6,7):

$$\ln s = -a_2 g^2 + a_4 g^4 - \mathrm{K} \approx -bD + b^2 C \qquad [1]$$

The coefficients a_n depend on the autocorrelation functions of molecular velocity and the applied pulse sequence. The last expression in [1] reflects the common interpretation of the signal in which *D* is the apparent diffusion coefficient and *C* is a function of diffusion time. The series [1] terminates at the first term in the special case of unrestricted diffusion in homogeneous medium only.

Equation [1] is generally valid for small b-factors. For large b-factors, the series start to diverge and the expansion becomes useless. The conventional bi-exponential form of the signal is a guess about the exact sum of this series at large b:

$$s = w_1 \exp(-bD_1) + (1 - w_1)\exp(-bD_2) \qquad [2]$$

This expression coincides with [1] in the limit of small b-factors. In this case the three parameter of [2] are combined in two independent combinations of [1]:

$$D = w_1 D_1 + (1 - w_1) D_2 \quad \text{and} \quad C = \tfrac{1}{2}(D_1 - D_2) w_1 (1 - w_1) \quad . \qquad [3]$$

The objective of this study can be formulated in this context as to analyse whether diffusion in the human brain falls in the regime of small or large b.

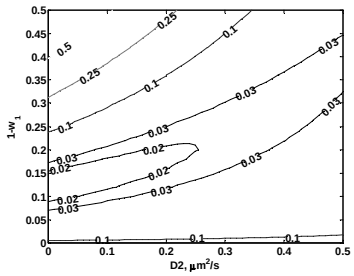

Fig.2. Contour map of χ^2 of fitted eq. [2]

Experiment. DWI measurements in a young healthy volunteer were performed on 3T Siement Trio MR scanner. TE/TR were 104/4000 ms, voxel size 4x4x5 mm^3, DE in slice direction, b_factor from 0 to 2500 mm^2/s in 16 steps, NA=4, scan time 12'04".

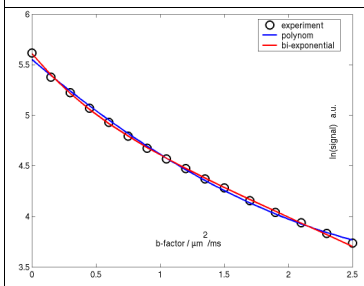

Fig.3. Data obtained in a cortical ROI

Results and Discussion. Fig. 1 shows ln *s* obtained in a ROI selected entirely in the white matter. The data are fitted with eq. [1] and eq. [2]. A good quality of both fits suggests that diffusion in macroscopically homogeneous tissue falls in the regime of small b-values. The presence of two diffusion coefficients cannot be proven by these data, since the bi-exponential function coincides with eq. [1] which follows from the first principles. The good quality of fit provided by eq. [1] agrees with previous results of paper (8) in which eq. [1] was used by other theoretical reasons. The redundancy of the three parameters as expressed by eq. [3] is illustrated in Fig. 2. It shows a contour map of the sum of squared deviations of data from eq [2] as a function of D_2 and 1-w_1 for a fixed value of D_1. This function has an almost degenerated minimum, which in the limiting case forms a valley described by eq. [3]. This can cause strong correlated fluctuations of results of fitting in such a way that the values of *D* and *C* are preserved.

Fig. 3 presents data which were obtained in a cortical ROI which includes grey matter, white matter and CSF. Data suggest a trend to a better fit quality by the bi-exponential function [2]. This is in agreement with the inhomogeneous content of the cortical ROI. Diffusion is truly bi-exponential in the presence of two physically distinguishable compartments. The size of each compartment should be much larger than the diffusion length, the condition which is hardly fulfilled for the typical cellular structures, but in each tissue type within a voxel or ROI.

To conclude, diffusion weighted signal in homogeneous tissues is described adequately by eq. [1] within the considered range of b-factors as suggested by theoretical reasons and experimental evidences. The presence of two exponential contributions can be justified in inhomogeneous regions which include macroscopic amount of different tissues.

References

(1) T Niendorf et al. MRM 36(1996)847. (2) RV Mulkern et al. NMR biomed 12 (1999) 51. (3) CA Clarc, D Le Bihan. MRM 44(2000)852. (4) AL Sukstanskii, DA Yablonskiy. JMR 157(2002)92. (5) AL Sukstanskii et al. MRM 50(2003)735. (6) PT Callaghan. Principles of Nuclear Magnetic Resonance Microscopy, Claredon Press 1991. (7) VG Kiselev et al. Introduction in Quantum Field Theory. Gordon and Breach 2000. (8) DA Yablonskiy et al. MRM 50(2003)664.