Nonlinear registration using variational principle for normalized mutual information and AOS scheme

P. Zhilkin[1], M. E. Alexander[1]

[1]National Research Council of Canada, Winnipeg, MB, Canada

We compare two multimodal nonlinear registration algorithms, based on Shannon Mutual Information (MI) and inverse Normalized Mutual Information (INMI) [1], incorporating a regularization term and fast numerical solver. This generalizes previous registration algorithm [2] for an arbitrary displacement field.

We apply a variational principle to a sum (1) of negative MI (similar to [6]), or in another case of INMI, and regularization term, resulting in diffusion equation (2) with respect to displacement field **u**

$$E(\mathbf{u}) = \begin{Bmatrix} -I(\mathbf{u}) \\ I_N(\mathbf{u}) \end{Bmatrix} + \frac{\alpha}{|\Omega|}\int d\mathbf{x}\,\psi\left(\sum_{j=1}^{m}|\nabla u_i|^2\right) \qquad (1)$$

$$\frac{\partial u_i}{\partial t} = V(\mathbf{x},\mathbf{u})\frac{\partial f_T(\mathbf{x}+\mathbf{u})}{\partial x_i} + 2\alpha\nabla\cdot(\psi'\nabla u_i) \qquad (2)$$

where α - regularizer weight, m – image dimensionality, $V(\mathbf{x},\mathbf{u}) = \iint d\xi d\eta\,\delta(\xi - f_T(\mathbf{x}+\mathbf{u}))\delta(\eta - f_R(\mathbf{x}))\left[\frac{p_\xi(\xi,\mu,\mathbf{u})}{p(\xi,\mu,\mathbf{u})} - \begin{Bmatrix} 1 \\ I_N(\mathbf{u}) \end{Bmatrix}\frac{p_{T\xi}(\xi,\mathbf{u})}{p_T(\xi,\mathbf{u})}\right]$,

I_N – INMI, f_R ,f_T – reference and test images, p – joint pdf of intensities of the image pair, p_T - marginal pdf for test image. We used regularizer $\psi(\cdot)$ described in [3], and the δ-function is approximated by a cubic B-spline. The diffusion equation is solved by a stable Additive Operator Splitting scheme [4].

A series of 30 2D test images were generated out of a single MR brain image (used as a reference image) by applying a smooth nonlinear displacement field with maximal displacement ~4.7 pixels, and adding independent 5% Rician noise to the image intensity. The intensity distribution of the test images was changed by applying a nonlinear intensity transformation [2]. Prior to registration the images were convolved with a low-pass filter. Registering the images first at coarse and then at successively finer resolutions was performed to reduce the risk of false matches. The coarseness of the filter depends on the maximal expected displacement between the images (the larger the displacement, the coarser the filter). All test images were successfully registered by both MI and INMI-based methods, achieving comparable MI values between the reference and registered images in most cases. An example of INMI-based registration is provided in Fig.1. One can notice an improved match, especially on the boundaries of features inside the brain. However, on the boundary of the skull the match is less pronounced. The value of MI between the test and reference images was 0.432 before registration, and 0.780 (0.778) after registration by MI (INMI) method. Distortion correction outside brain was not expected because of lack of features in these areas. Increasing the weight of the regularizer in (1) usually results in a smoother displacement field and smaller achieved MI values.

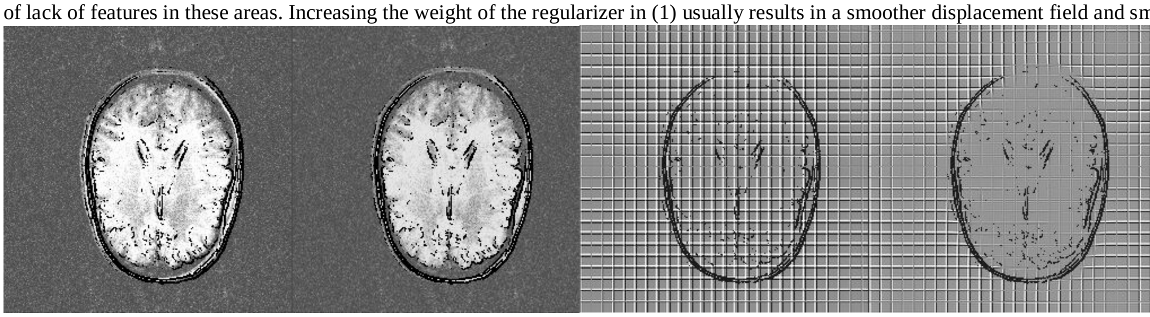

Fig.1 INMI registration of a 2D MR brain image. From left to right: 1. unregistered test image, 2. registered test image, 3. difference between a grid (grid 0) and the same grid distorted by the displacement field used to create the test image (grid 1). 4. difference between grid 0 and the grid 1 distorted by the displacement field found by the registration algorithm; ideally these two grids should coincide. All images are overlaid with the contour of the reference image.

In another example, a pair of 3D tomato MR images was considered. After the first (reference) image was acquired, the tomato was squeezed to introduce a local nonlinear distortion field, and another (test) image was acquired. The intensity of the test image was modified in order to change its modality [2]. First the image pair was registered affinely using the patch algorithm [5], to correct for possible global motion. Then both MI- and INMI-based methods were employed to correct for nonlinear distortions. Both algorithms successfully registered the image pair, achieving comparable MI values between the registered and reference images; the INMI results are shown in Fig.2. The value of MI between images before registration was 0.566, after affine registration 0.640, and after registration by MI and INMI methods 0.710 and 0.711, respectively.

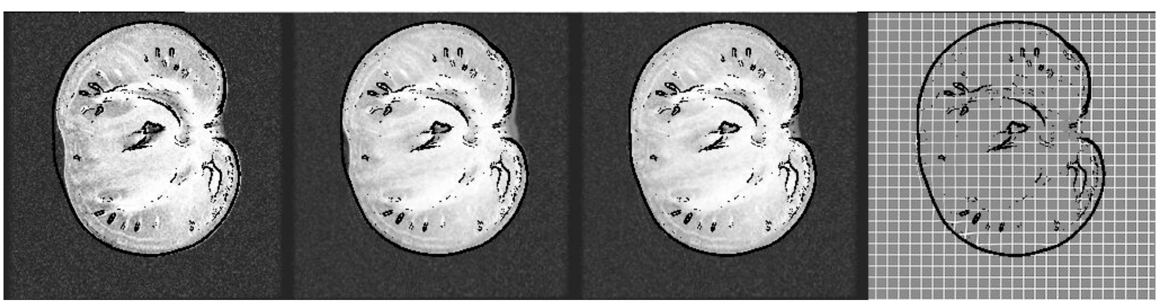

Fig.2. INMI registration of 3D image. A single slice is shown. From left to right: 1. unregistered test image, 2. affinely registered test image, 3. registered test image, 4. a grid distorted by affine and nonlinear motion found by registration; all images are shown with a contour of the reference image.

Conclusion. Both MI and INMI methods were able to properly register (with a similar quality, comparable final MI values between the registered and reference images, and comparable CPU time) a series of artificial 2D images against an image of different modality. The methods also properly registered a pair of 3D images.

The authors thank Dr. L.Ryner of National Research Council Canada for providing MR image of a tomato used in this study.

References: **1.** Studholme et al, *Pattern Recognition*, 32: 71-86, 1999 **2.** Alexander et al, *Proc. SPIE Int. Soc. Opt. Eng.* 5032: 1053-1063, 2003 **3.** Weickert et al, *J. Math. Imag. And Vision* 14: 245-255, 2001 **4.** Weickert et al, *IEEE Trans. Imag. Proc.* 7(3): 398-410, 1998 **5.** Zhilkin et al, *Magn. Reson. Imag.* 18: 1143-1150, 2000 **6.** Hermosillo et al, *Int. J. of Comp. Vision* 50:329-343, 2002