

# Analytical Parameter Estimation in Arterial Spin Labeling Time Series by Fourier Transformation

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**Introduction:** Arterial spin labeling (ASL) can be used to quantify perfusion without the use of contrast agents. A major difficulty in obtaining quantitative perfusion values is that, in principle, the exact time when the labeled blood reaches the capillary exchange site (the so-called bolus arrival time BAT) needs to be known for each imaging voxel. Besides improving the accuracy of perfusion measurements the knowledge of BAT might reveal valuable additional patho-physiologic information, e.g., of strokes or tumor vascularity. Time series of ASL images with different inflow times TIs are typically fitted to a model function using a non-linear least-square fitting algorithm. The non-analytical nature of the optimization step can cause instabilities and unrealistic parameter values for signal behavior where the start parameters are not set properly (e.g. arteries). In this work, we present a technique that yields accurate estimates of the most important parameters (perfusion and BAT) without using numerical optimization schemes. It will be shown that a Fourier transform of the time series provides accurate information on perfusion and BAT for both, macro- and micro-vascular signal curves.

**Material and Methods:** The signal difference of labeled and non-labeled ASL times series can be modeled as follows ([1]):

$$dM(t) = Heaviside(t - BAT) \cdot 2 \cdot \frac{M_0 \cdot f \cdot e^{-t \cdot R_{1a}}}{(R_{1a} - R_{1app})} \cdot (e^{-(t-BAT)(R_{1app} - R_{1a})} - 1), \quad (Eq.1)$$

where  $M_0$  describes the equilibrium magnetization of a voxel containing blood spins only,  $f$  the local perfusion and  $BAT$  is the bolus arrival time of the tagged bolus.  $R_{1a}$  and  $R_{1app}$  are the longitudinal relaxation rate of arterial blood and perfused tissue (as measured by slice-selective inversion recovery), respectively.

For  $t > 0$  the Fourier transform of this model function is:

$$d\hat{M}(\omega) = Fourier(dM(t)) = 4 \cdot M_0 \cdot f \cdot \frac{R_{1a} \cdot R_{1app} - \omega^2}{(R_{1app}^2 + \omega^2) \cdot (R_{1a}^2 + \omega^2)} \cdot e^{-BAT \cdot R_{1a}} \cdot e^{-I \cdot BAT \cdot \omega} \quad (Eq.2)$$

Therefore, the constant part ( $\omega=0$ ) is described by:

$$d\hat{M}(0) = 4 \cdot \frac{M_0 \cdot f}{R_{1app} \cdot R_{1a}} \cdot e^{-BAT \cdot R_{1a}}, \quad (Eq.3)$$

which is proportional to the perfusion  $f$  but still depends on the  $BAT$ . However, the bolus arrival time can be estimated by using the phase information. The phase  $\phi$  is:

$$\phi(d\hat{M}(\omega)) = BAT \cdot \omega \quad (Eq.4)$$

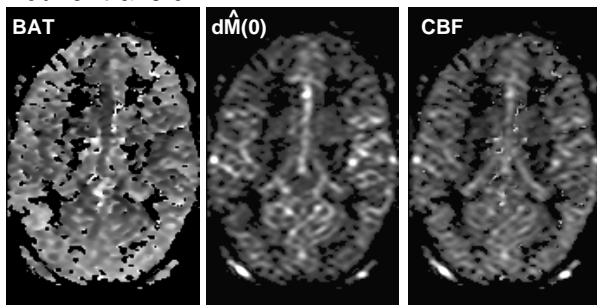
In principle,  $BAT$  could be estimated from each Fourier component. In practice, the component with the smallest non-zero value of  $\omega$  will be used, because of the best signal-to-noise ratio (SNR), i.e.  $\omega = 2\pi/t_{MAX}$  with  $t_{MAX}$  being the latest inflow time sampled.

For parameter estimates a time series acquired with a single-shot 3D-GRASE sequence [2] with 29 different inflow times ranging from 200 ms to 3000 ms in 100 ms increments was used. 24 slices with isotropic resolution of 4 mm could be acquired within a total measurement time of 17 min. Perfusion and  $BAT$  values were estimated on a pixel basis by a non-linear least-square fit (Levenberg-Marquardt) as well as by using the proposed Fourier transform.

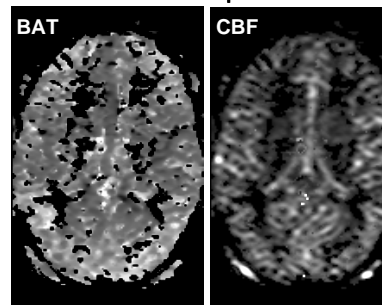
**Results:** Figure 1 shows the result of the parameter estimates for both, the least square optimization and the Fourier transform. The parameter maps were masked by a thresholding the perfusion value.  $BAT$  estimated by both methods are very similar. In the Fourier technique signal intensity of vessels with short  $BAT$  are pronounced if no correction according to Eq.3 is performed. After correction the resulting CBF is comparable to the least square estimate. Some artificially high values arise from pixels with very low SNR where a long  $BAT$  resulted from the Fourier transform.

**Discussion and Conclusion:** The results show that the arrival time  $BAT$  of the labeled blood can be estimated by using the Fourier transform of an ASL time series. CBF can be calculated by using the Fourier transform for  $\omega=0$  and correcting for different  $BAT$  using Eq. 3. The advantage of this method is that it is an analytical solution compared to the optimization scheme typically used. High frequency noise is not present at the small value of  $\omega$  used for parameter estimation. One drawback might be that in the current implementation the times series has to be sampled equidistantly but this could be improved by using an adequate Fourier transform algorithm. We presented an easy and robust implementation for parameter estimation in ASL time series which avoids fit parameter adjustments by using an analytical algorithm.

## Fourier transform



## Nonlinear least-square fit



## References:

- [1] R. B. Buxton, et al., *Magn Reson Med* (1998), **40**: 383-96.
- [2] M. Günther, et al., ISMRM 2004, Kyoto, Japan, p.714

Fig.1: Estimation of CBF and BAT using Fourier transform (left) and nonlinear least square fit (right).