Lossy dielectric sphere in the multi-channel excitation birdcage

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Overview

Fairly recently, studies[1]-[3] of the dielectric effects for the transverse RF field of the shielded birdcage coil in the presence of a lossy dielectric sphere have indicated large B_1 non-uniformity characteristics across the entire spherical phantom. For these solutions, it is assumed that the birdcage coil generates a constant transverse B_1 field across the sample that is generated by a $\cos\varphi$ current distribution. The non-uniformity patterns of the B_1 field inside the sphere are the results of the interaction between the RF field and the lossy dielectric of the spherical object. It is, however, intriguing to understand the behavior of the RF field propagation inside a dielectric sphere where the current density of the birdcage coil follows other than the traditional $\cos\varphi$, $\sin\varphi$ azimuthal behavior. In particular, in this paper, the solution for a 3D source of an electromagnetic field acting upon a spherical dielectric phantom with non-zero conductivity ($\varepsilon\approx60$, $\sigma\approx0.7$ S/m) is presented with the intention to study the effects of RF field homogeneity in spherical phantoms at high frequencies. In order to obtain analytical expressions, spherical vector harmonics (SVH) are introduced, where the current density of the cylindrical current source is considered to have a $\cos m\varphi$ or $\sin m\varphi$ azimuthal distribution. Preliminary results indicate very good agreement of this theoretical model with published results [3], [4] regarding the B_1 field behavior inside a dielectric sphere when the azimuthal current distribution of the birdcage coil has a simple sinusoidal ($\cos\varphi$, $\sin\varphi$) behavior.

Theory

An infinitely long and shielded birdcage-like structure with the ideal $\cos m\varphi + i\sin m\varphi$ azimuthal distribution of the current is assumed. The radius of the birdcage is taken to be R_B cm and the radius of the shield is R_S. To calculate the field inside the birdcage, we implement a full 3-D wave solution. The vector potential of a current source in the air is given by $\mathbf{A}_m(r,\varphi,z,t) = \hat{z}A_m J_m(\lambda r)e^{im\varphi}e^{i\omega t}e^{ihz}$, where A_m is a constant depending on boundary conditions, the position of the sources, and the radius of the shield, $J_m(\lambda r)$ is the Bessel function, and $k^2=\lambda^2+h^2=\mu_0\varepsilon_0\omega^2$ is the dispersion relation in the vacuum (≈air), while its expression inside the sphere is $k^2 = i\mu_0\omega\sigma + \mu_0\varepsilon_0\varepsilon\omega^2$. The corresponding magnetic field will be

$$\mathbf{B}(r,\varphi;m) = \nabla\times\mathbf{A}(r,\varphi,z;m) = A_n e^{im\varphi}e^{ihz}e^{-i\omega t}\left(\frac{im}{r}J_m(\lambda r)\hat{r} - \frac{d}{dr}J_m(\lambda r)\hat{\varphi}\right) \quad (1)$$

where $\hat{r}, \hat{\varphi}$ are unit vectors in cylindrical coordinates.

In order to determine the behavior of the RF field inside a spherical phantom, spherical coordinates [5] are desired, and the **B** field from Eq. (1) needs to be expressed in terms of spherical vector harmonics (SVH). Considering the definition of SVH as [6]:

$$\mathrm{Y}_{lm}^{(e)} = \frac{1}{\sqrt{l(l+1)}}\nabla_{\mathbf{n}}Y_{lm}, \quad \mathrm{Y}_{lm}^{(m)} = \frac{1}{\sqrt{l(l+1)}}\mathbf{n}\times\mathrm{Y}_{lm}^{(e)}, \quad \mathrm{Y}_{lm}^{(r)} = \hat{R}Y_{lm}, \quad (2)$$

where the scalar spherical harmonics $Y_{lm} = Y_{lm}(\theta,\varphi)$ and vector operator $\nabla_{\mathbf{n}}$ are defined as

$$Y_{lm}(\theta,\varphi) = (-1)^{\frac{m+|m|}{2}} i^l \sqrt{\frac{2l+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}P_l^{|m|}(\cos\theta)e^{im\varphi}, \qquad \nabla_{\mathbf{n}} = \hat{\theta}\frac{\partial}{\partial\theta} + \hat{\varphi}\frac{1}{\sin\theta}\frac{\partial}{\partial\varphi}. \quad (3)$$

and $\hat{R}, \hat{\theta}, \hat{\varphi}$ represents unit vectors in spherical coordinates and $P_l^m(x)$ is the associated Legendre polynomials.

For the analytic derivation of the solution, it is considered that the source of the EM field has the wave vector $\mathbf{k}$ perpendicular to the main axis of the birdcage coil. Thus Eq. (1) can be rewritten for each m as

$$\mathbf{B}(R,\theta,\varphi;m) = \sum_{n=0}^{\infty}\left(B_{nm}^{(e)}\mathrm{Y}_{nm}^{(e)} + B_{nm}^{(m)}\mathrm{Y}_{nm}^{(m)} + B_{nm}^{(r)}\mathrm{Y}_{nm}^{(r)}\right) \quad \text{with} \quad (4)$$

$$B_{nm}^{(e)}(R) = \int\mathbf{B}(R,\theta,\varphi;m)\mathrm{Y}_{nm}^{(e)*}d\Omega, \quad B_{nm}^{(m)}(R) = \int\mathbf{B}(R,\theta,\varphi;m)\mathrm{Y}_{nm}^{(m)*}d\Omega, \quad B_{nm}^{(r)}(R) = \int\mathbf{B}(R,\theta,\varphi;m)\mathrm{Y}_{nm}^{(r)*}d\Omega. \quad (5)$$

Having the components of the external magnetic field (and through Maxwell equations the electric field), and considering the same kind of expansion in SVH inside the sphere and for the scattering wave outside, the unknown coefficients from boundary conditions for normal and tangential components of the electric and magnetic field can be derived.

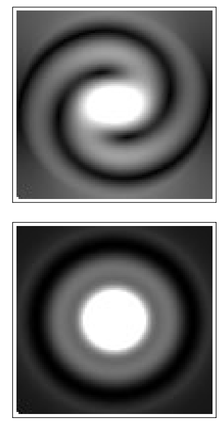

Fig.1. Real and absolute values of the B_1 field at z=0.

Results and Discussion

As an example of this analytic work, the behavior of the RF magnetic field of the birdcage resonation with a spherical phantom at 400 MHz is investigated. For the birdcage coil, its inner radius is set to be R_B =0.146 m and its shield is set to be R_S.=0.176m. A spherical phantom with radius of 0.,0925m is located and centered inside the birdcage coil. A current distribution with a simple sinusoidal ($\cos\varphi$, $\sin\varphi$) behavior for the birdcage coil is considered. Fig. 1 represents the real and the magnitude image of the RF field propagation inside the spherical phantom. The characteristics of bright and dark rings in figure 1 indicating the strength of the fields at these points and are very similar to the ones published by Tropp [3]. Furthermore, half of the B1 profile cross section at the center of the sphere is shown in Fig. 2, which is very good agreement with the results presented by Tropp [3].

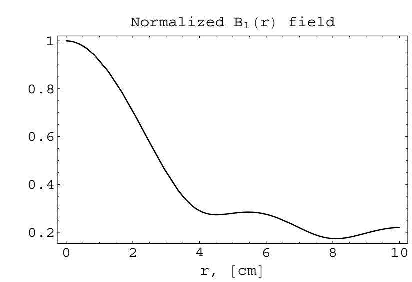

Fig. 2. Radial dependence of the B_1 field at z=0.

Conclusion

An analytical methodology to predict the behavior of the RF transverse field of the birdcage coils using currents with higher harmonics in a spherical phantom has been presented. Preliminary results considering simple sinusoidal current density of the birdcage at 400 MHz are in good agreement with previous methodologies.

References

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