An Efficient General Purpose Electric Field Calculation Technique for Gradient Fields in MRI

R. E. Feldman[1], J. W. Odegaard[2], W. B. Handler[2], B. A. Chronik[2]

[1]Medical Biophysics, University of Western Ontario, London, Ontario, Canada, [2]Physics and Astronomy, University of Western Ontario, London, Ontario, Canada

Introduction: During MRI, subjects are exposed to magnetic gradient fields that switch at frequencies of approximately 1 kHz. In some cases subjects have experienced peripheral nerve stimulation (PNS) due to exposure to these fields. The switched magnetic field induces electric fields on the order of tens of volts per meter within the scanner. The field causing PNS is hypothesized to be the combination of the electric fields due to the coil and to the charge built up on the surface of the tissue in response. Previous work in developing methods for modeling the total resultant electric fields include fast and efficient analytic methods[1], which suffer from an inability to consider anatomically accurate geometries, and general full-wave FDTD solutions[2], which suffer from long computation times for high resolution solutions at low frequencies. The development of gradient coils designed to reduce the magnitude of the PNS effect require a modeling method that can handle general geometries in a computationally efficient manner. In this abstract, we present a simple finite-difference method designed to meet these criteria.

Methods: Electric field is calculated by summing contributions of two components: the scalar potential (ϕ), and the vector potential (A):

(a) $E_{tot} = -\frac{\partial A}{\partial t} - \nabla\Phi$ (b) $A(x,y,z) = \frac{\mu_0}{4\pi}\int_{V'} \frac{J'(x',y',z')e^{-jkR}}{R} dV'$ c) $\Phi(x,y,z) = \frac{1}{4\pi(\sigma + j\omega\varepsilon)}\int_{V'} \frac{\rho'(x',y',z')e^{-jkR}}{R} dV'$

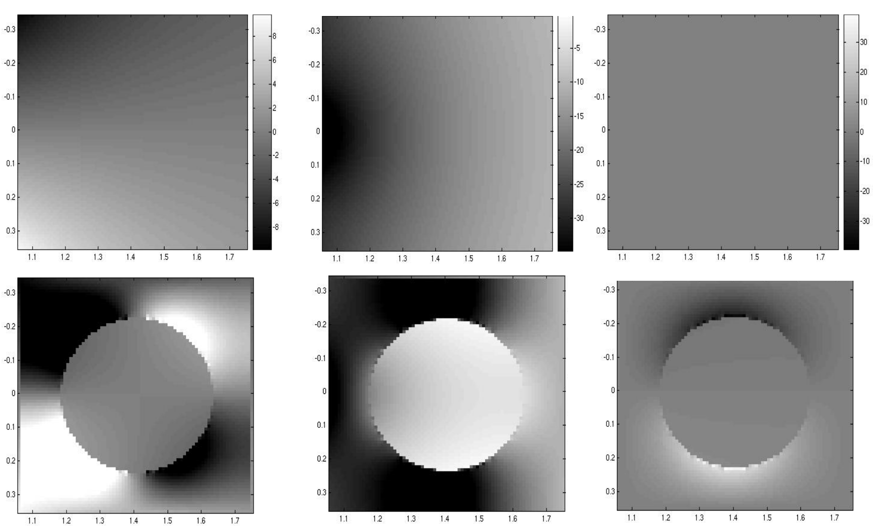
In order to simplify these equations a quasi-static approximation is applied [3]. This assumes that the wavelength and skin depth are much larger than the dimensions of the coil and that the tissue is purely resistive. The vector potential is calculated directly from the wire pattern of the gradient coil: $A(x,y,z) = (\mu_o I/4\pi)\int(dl'/R)$. The scalar potential is determined by applying Laplace's equation, $\nabla^2\phi=0$, and the boundary conditions $d\phi/dn = n\cdot E_a$.

The electric field for the empty coil is calculated first, obtained by taking the time derivative of the calculated vector potential for that coil. A finite-difference method [4] is then used to solve Laplace's equation to obtain the scalar potential. For all interior voxels, the sum of the scalar potential in each of the six neighbors equals six times the value of the potential in the central voxel. At the surface of the object, we treat the voxels differently, each weighted according to the square of the magnitude of the appropriate normal component (n_x, n_y, or n_z). The three weighted neighboring scalar potential components(Φ_1, Φ_2, and Φ_3) just inside the boundary are summed and the potential of the boundary voxel (Φ_0) subtracted from it. The scalar potential of the boundary voxel is then adjusted so that that the value of this equation is equal to the value of the electric field (E_x, E_y, E_z) projected onto the vector normal to the boundary at that point.

(d) $n_x^2\bullet\Phi_1+n_y^2\bullet\Phi_2+n_z^2\bullet\Phi_3-\Phi_0 = E_x\bullet n_x + E_y\bullet n_y + E_z\bullet n_z$

To calculate the boundary conditions for general objects, a three dimensional voxelated image of the object is needed. Each voxel in the object is tested to determine if it is on the surface. The normal at each boundary point is then calculated by taking the cross products between the voxel being evaluated and eight neighboring surface points. The cross products are adjusted, by multiplying by -1 as needed, so that they all point from inside the object to outside. The vectors are then averaged. The resulting vector is the normal used to set the boundary conditions for that point, as described above. It is important to note that with this method, the set of normal vectors for any geometry only need to be evaluated once. They can then be stored and used for modeling the electric fields induced within and around that geometry for any gradient coil.

Results and Discussion: To demonstrate the method, we consider a calculation for a conductive sphere of radius 27.7 cm placed within the electric field produced by the time-varying magnetic field of a 1 m radius solenoid. The sphere was placed 1.4 m along the x-axis (i.e. outside and to the right of the solenoid). A cubic volume with an edge width of 70 cm (77 voxels) surrounded the sphere. The dataset was 456,533 voxels and using a 1.4 GHz processor, the algorithm took less than 5 minutes to compute the normals and converge on a solution for the scalar potential and total electric field. The results are shown in the figure. Note that the colour bars have been re-scaled for each of A,B, and C as shown.



A B C

Electric field due to 1 meter radius solenoid centered at (0,0,0) (top) the electric field resulting from distortion caused by a sphere(bottom) a)field in x-direction b)field in y-direction c)field in z-direction

We have also successfully used this method in conjunction with the visible man head data set [5] to compute the total electric fields a person would be exposed to within a head-sized gradient coil. This technique is fast and yet general enough to allow us to model the effects that a whole series of possible gradient coils would have on various human geometries during MR scanning.

[1] M. Bencsik, et al. Phys Med Biol 47:557-76 (2002)
[2] C.M. Collins, et al. Proc. ISMRM, 12:661 (2004)
[3] B.J. Roth, et al. EEG Suppl. 43:268-278 (1991)
[4] W.H.Press,S.A.Tukolsky. Numerical Recipes in C++, 871 (2002)
[5] US Air Force: www.brooks.af.mil/AFRL/HED/hedr/hedr.html