Stream Function Method for Design of Arbitrary-Geometry Shielded Gradient Coils

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Introduction

This work is a continuation of our research published in [1]. The paper described a new stream function method to design gradient coils. Using this method we are able to determine the current distribution to achieve a certain prescribed magnetic field distribution in the Region of Interest (ROI). A cost function introduced in [1] can account for field linearity, magnetic energy and Lagrange multipliers if the gradient coil has to be self-balanced.

Over the past several years a variety of theoretical design methods for the construction of gradient coils have been developed. In paper [2] D. Green *et al.* minimize a weighted combination of power, inductance, and the square difference between actual and desired field. Representing the current as a Fourier series they find optimal coefficients that minimize the cost function.

As an alternative, in this paper we describe a new approach for the coil design that is largely independent of the shape of the current-carrying surface. We will demonstrate the success of our approach by designing a shielded crescent G_x gradient coil.

Theory

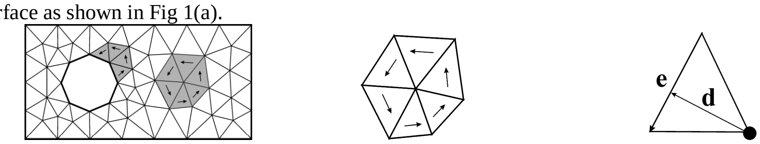
As mentioned above, a cost function Φ is introduced in the form

$$\Phi = \frac{1}{2}\sum_{k=1}^{K} W(\mathbf{r}_k)\left(B_z(\mathbf{r}_k) - B_{des,z}(\mathbf{r}_k) + B_{off,z}\right)^2 + \alpha W_{magn} + \beta\frac{1}{2A_s}\int_{S_s}(\mathbf{B}\cdot\mathbf{n})^2\,dS_s - \lambda_x M_x - \lambda_y M_y - \lambda_z M_z \tag{1}$$

where $W(\mathbf{r})$ is a weight function, $B_{des,z}(\mathbf{r})$ is the *z*-component of the desired magnetic field, W_{magn} is magnetic energy of the current, and α is a weight coefficient. Also, $\mathbf{B}$ is the total magnetic field, S_s is the surface of the shield, M_x, M_y, M_z are the components of the torque vector $\mathbf{M}$ which are calculated with respect to the origin as a fixed point. In (1), the first term is the square deviation of the magnetic field from the prescribed one, the second term is the magnetic energy of the coil, and the third term measures the magnetic flux through the shield. The remaining terms are Lagrange multipliers that ensure the coil to be self balanced. Expressions for W_{magn} and $\mathbf{M}$ can be cast in the form

$$W_{magn} = \frac{\mu_0}{8\pi}\int_S\int_{S'}\mathbf{J}(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,dSdS', \quad \text{and} \quad \mathbf{M} = \int_S \mathbf{r}\times\left(\mathbf{J}(\mathbf{r})\times\mathbf{B}_0(\mathbf{r})\right)dS \tag{2}$$

This formulation allows us to model a surface as shown in Fig 1(a).



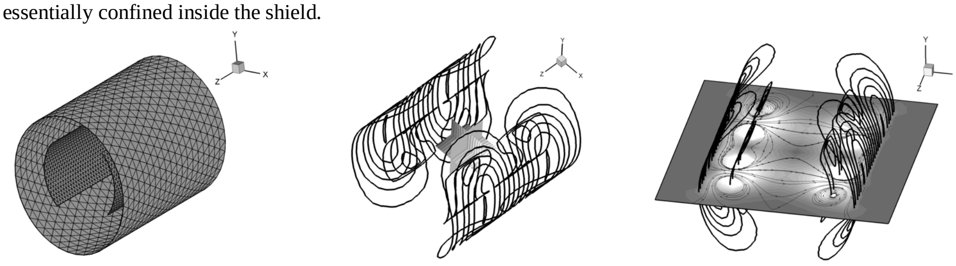
(a) (b) (c)

Figure 1: (a) Surface discretization, (b) current flow convention, (c) basis function.

We associate a current with each node as shown in Figs. 1(b) and (c). For each triangle belonging to a chosen node we define a basis function as $\mathbf{f}(\mathbf{r}) = \mathbf{e}/(|\mathbf{e}||\mathbf{d}|)$. All basis functions are assumed to rotate in the same (clockwise or counterclockwise) direction. The surface current can then be approximated as a combination of rotational currents. Furthermore, we express $B_z(\mathbf{r})$ and W_{magn} and $\mathbf{B}$ in terms of currents I_n and obtain a system of linear equation for I_n, the offset field $B_{off,z}(\mathbf{r})$, and for λ_x, λ_y, λ_z.

Example of coil design

To demonstrate how the algorithm works, we consider a primary coil composed of two plates, each of which has a size of 20×10 cm. These plates are curved with a radius of *R*=6.5 cm and positioned side-by-side. The primary coil is placed inside the cylindrical shield (secondary coil). This arrangement is next discretized into a triangular mesh as seen in Figure 2(a). If we require $R = 1\Omega$ as primary coil resistance then each groove of the primary coil in Fig. 2(b) contains 9 turns of AWG-20 wire. Also, each groove of the secondary coil contains one AWG-20 wire resulting in 0.3 Ω resistance. The gradient strength in the center of ROI is 23.9 G/cm for a drive current of 100A. The total coil inductance is found to be $L = 227\,\mu\text{H}$. Introduction of a shield for a gradient coil results in a higher total resistance and lower gradient strength. As a benefit we obtain a lower inductance and the shielding effect. As we see from Figure 2(c), the magnetic field is very low outside of the shield; the magnetic field lines are essentially confined inside the shield.



(a) (b) (c)

Figure 2: G_x crescent coil design approach. (a) mesh, (b) wire pattern and *z*-component of magnetic field, (c) wire pattern and absolute value of magnetic field.

Conclusion

This paper presents a novel version of the stream function method for the design of single- and multi-surface gradient coils. It is shown that shielding can be successfully introduced by adding an extra term to the cost function.

[1] R.A. Lemdiasov, R. Ludwig, C. Ferris, "Stream Function Method for Design of Arbitrary-Geometry Gradient Coils," Proc. Intl. Soc. Mag. Reson. Med. 12, 2004.
[2] D. Green, R.W. Bowtell, P.G. Morris, "Uniplanar Gradient Coil for Brain Imaging," Proc. Intl. Soc. Mag. Reson. Med. 10, 2002.