Slack, Minimum Inductance Gradient and Shim Coil Design

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Introduction

Magnetic resonance imaging (and spectroscopy) requires a highly homogeneous, strong magnetic field to generate undistorted images (and narrow line-spectra). Shim coils are used to homogenise the field and gradient coils are used to encode the NMR signal spatially. Both types of coil are designed to generate magnetic fields formed from spherical harmonics. The coils must also be of low inductance so currents in them can be switched on and off rapidly. The inductance and homogeneity of such coils are inversely related in an undetermined manner. Methods of designing coils by trading off some homogeneity for lower inductance have been presented before [1,2]. Outlined here is a more controllable technique that works by finding the length-constrained coil of minimum inductance that generates a magnetic field variation falling within acceptable tolerances.

Methods

The coils were constrained to have a half length, l, using the method of Carlson *et al.* [1], in which the current density is composed from a sum of N weighted, truncated sinusoids with spatial frequencies that are a multiple of π/l. In general, 9 harmonics were used, except for even order zonal coils (Z0, Z2, Z4 etc.), for which the 0th harmonic was also used. The inductance, L, of a current distribution, defined by the azimuthal component, $j_\phi(\phi,z)$, flowing on the surface of a cylinder of radius a, is given by Turner [2] as $L = \frac{-\mu_0 a}{I^2}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}\left|j_\phi^m(\chi)\right|^2 I_m^{'}(\chi)K_m^{'}(\chi)d\chi$, (where $j_\phi^m(\chi)$ is the two-dimenseional Fourier transform of $j_\phi(\phi,z)$ and $I_m(\chi)$ and $K_m(\chi)$ are the modified bessel functions). An equation for the z-component of the magnetic field B_z can also be derived [3] $B_z = \frac{-\mu_0}{2\pi a}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty} j_\phi^m(\chi)K_m^{'}(\chi)I_m\left(\chi\rho/_a\right)e^{im\phi}e^{i\chi z/_a}\chi d\chi$. In the process of coil design, the inductance was minimised within specified homogeneity constraints to give a harmonic coefficient weighting for each sinusoid. A target field was defined by assigning field values (using the appropriate spherical harmonic), B_q, at Q points, each with an associated maximum allowed inhomogeneity (or slackness), δ_q. The slackness constraints were defined by the inequality $E_{Bq} = \left|\frac{B_z(\mathbf{r}_q)-B_q}{B_q}\right| \le \delta_q$, where $B_z(\mathbf{r}_q)$ is the z-component of the magnetic field at the qth target field point. It is most intuitive to define the slackness as a percentage of the maximum value of B_q, but it may be given any value at any point. The coil resistance may be included in the minimisation calculation by adding it to the inductance with a weighting factor.

The inequality-constrained minimisation was carried out using sequential quadratic programming within the Matlab® Optimization Toolbox function `fmincon`. The harmonic coefficients were then used to construct the stream function of the current density, the equally spaced contours of which give the wire positions for the coil design. It was found that the performance of a Z2 and Z4 coils could be greatly improved by adding a variable Z0 offset to the target field. This was negative and dependent on the parameters of the coil design.

η^2/L provides a measure of the performance of the coil that is independent of current and coil diameter. η characterises the amount of the desired spherical harmonic generated by the coil, and can be calculated using a small z and ρ expansion of the expression for B_z [2]. Currently used shim coils are generally composed of small numbers of loop and saddle arrangements using methods outlined by Roméo and Hoult [4]. The η^2/L values of these discrete coils were calculated (using a wire thickness of 0.04a m) to compare with the performance of coils designed using the new method. The field variation was also calculated and the size of the largest cylindrical volume within which the field inhomogeneity was less than 2% was measured. These dimensions were then used to define the extent of the target field grid for designing distributed coils, with the same homogeneity over the same volume using 2% inhomogeneity-constrained inductance minimisation.

Results

Table 1 shows the η^2/L comparisons for some discrete gradient and low order shim coils and the equivalent coils designed using the new method. 44 target points were defined over 4 target radii and 11 axial points equally spaced within the volume defined by the 2% homogeneous regions of the discrete coils. The half-length, l, of the coils was set to 2a throughout for consistency (longer or shorter coils may alter η^2/L slightly). Fig. 1 shows two examples of coils designed using this method.

Coil Type	Discrete η^2/L $[(T^2m^{-n})^2H^{-1}]$	2% Homogeneous Area ($\rho \times z$) [m]	Slack η^2/L $[(T^2m^{-n})^2H^{-1}]$
Z0	$2.3 \times 10^{-8}\ a^{-3}$	$0.28a \times 0.27a$	$10.9 \times 10^{-8}\ a^{-3}$
Z1	$5.5 \times 10^{-8}\ a^{-5}$	$0.61a \times 0.43a$	$12.5 \times 10^{-8}\ a^{-5}$
Z2*	$1.4 \times 10^{-8}\ a^{-7}$	$0.51a \times 0.73a$	$7.1 \times 10^{-8}\ a^{-7}$
Z3	$1.2 \times 10^{-8}\ a^{-9}$	$0.55a \times 0.60a$	$4.6 \times 10^{-8}\ a^{-9}$
X	$3.1 \times 10^{-8}\ a^{-5}$	$0.44a \times 0.45a$	$8.3 \times 10^{-8}\ a^{-5}$
XZ	$1.9 \times 10^{-8}\ a^{-7}$	$0.49a \times 0.64a$	$4.2 \times 10^{-8}\ a^{-7}$
X2-Y2	$3.4 \times 10^{-8}\ a^{-7}$	$0.57a \times 0.39a$	$7.8 \times 10^{-8}\ a^{-7}$

Table 1. *Results of the comparison of η^2/L measurements for discrete and 2% slack coils. (* Target field offset of -0.90 was used)*

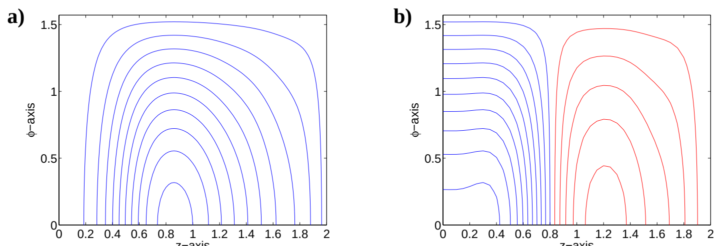

Figure 1. *Octants of wire paths for a) an X gradient coil and b) a ZX coil, designed to have a 2% slackness over the regions in table 1 (red colour denotes current flowing in the opposite sense to blue).*

Conclusion

This coil design method provides a significant reduction in inductance with a high degree of control over the field homogeneity. The results in Table 1 show an improvement of the η^2/L measure for 2% slack coils over that of discrete wire coils (this is also the case when comparing coils at other levels of inhomogeneity). The inductance at fixed efficiency is reduced by 2 to 5 times using this approach. When the resistance is added into the calculation, there is little effect on the resulting coil designs. The calculated η^2/L values for the discrete coils agree well with values of experimentally measured values for real coils. The slack approach offers an excellent method of finding the optimal gradient coil design for a given homogeneous volume. The shim coils designed using this approach have considerably lower inductance than conventional discrete coils, which is important for the method of dynamic shimming.

References

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