High-Pass Two-Dimensional Ladder Network Resonators

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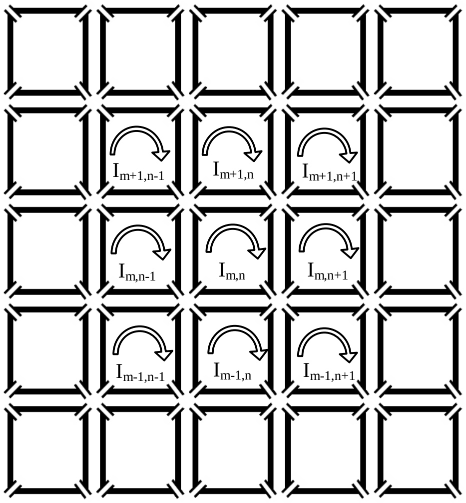

Figure 1 - A 5x5 square mesh high-pass two-dimensional ladder network. The next-to-highest eigenvalue corresponds to a normal mode giving rise to B_1 fields with good spatial homogeneity above the resonator plane.

Introduction: Multi-element radiofrequency resonator designs are currently of interest for magnetic resonance imaging due to their superior sensitivity for anatomical structures near the surface of the body as well as their utility for parallel imaging applications. In this abstract a solution is presented for the general problem of an inductively coupled two-dimensional resonator array in both the strong coupling "eigenmode" limit, and the weak coupling "phased array" limit. It is demonstrated that in the strong coupling limit the array produces a high-frequency resonant mode that can be used to generate the traditional quadrature B_1 field, and in the limit of weak or zero coupling reduces to the familiar phased array suitable for parallel imaging applications. The theory is compared to experimental results from two prototype resonators operating at 128 MHz. The goal of this work was to gain an understanding of the underlying physics of these structures for applications to high-field MRI.

Theory: The general problem of two-dimensional ladder network resonators has been shown to be closely related to the problem of a vibrating mechanical membrane with suitable boundary conditions. The problem is most easily solved by writing down a recursion relation for Kirchhoff's voltage equations on the meshes of the structure of interest. The dispersion relation for the eigenvalues yields the frequency spectrum, while the eigenfunctions represent the mesh current amplitudes. From the eigenfunctions it is straightforward to calculate B_1 maps.

Low-pass two-dimensional ladder networks have been described previously, and offer a doubly degenerate homogeneous mode for circularly polarized magnetic resonance imaging applications *(1)*. However, higher field applications of these structures are limited by the fact that the eigenvalue of the most homogeneous normal mode is lowest in frequency. The high-pass analog to the low-pass two-dimensional ladder network resonator is a collection of inductively coupled resonators as shown in Fig. 1. Each element is represented by four conducting strips having a self inductance L, joined by a capacitor C at each corner. The recursion relation for this structure can be written as

$$(\omega L - 1/\omega C)\, I_{m,n} - \kappa\,\omega L\,(I_{m+1,n} + I_{m,n+1} + I_{m-1,n} + I_{m,n-1}) - \alpha\,\omega L\,(I_{m+1,n+1} + I_{m-1,n+1} + I_{m-1,n-1} + I_{m+1,n-1}) = 0\,, \qquad [1]$$

where the constants κ and α represent the coefficients of mutual inductance between nearest neighbors sharing a common leg and those on the diagonal respectively. The $I_{m,n}$ are the current amplitudes in the m,nth element. Trial eigenfunctions of the form $I_{m,n}(\Omega,\Gamma) = A_{\Omega,\Gamma} \sin(m\pi\Omega/(M+1))\sin(n\pi\Gamma/(N+1))$ yield the following dispersion relation:

$$\omega^2 = (LC)^{-1}(1 - 2\kappa(\cos(\pi\Omega/(M+1)) + \cos(\pi\Gamma/(N+1))) - 4\alpha\cos(\pi\Omega/(M+1))\cos(\pi\Gamma/(N+1)))^{-1}. \qquad [2]$$

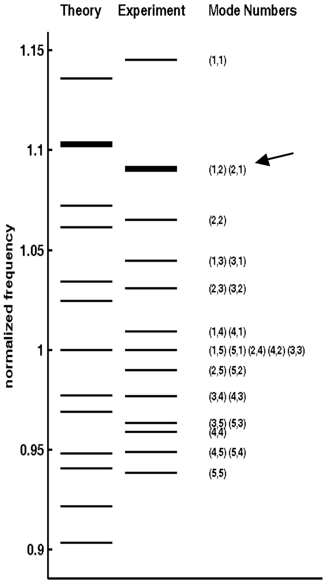

Figure 2 - Theoretical and experimental eigenvalues for the 5x5 inductively coupled planar array. The eigenvalue for the (1,2)(2,1) doublet (arrow) yields a mode suitable for quadrature operation that produces a B_1 field of high spatial homogeneity. The frequency scale is normalized to the single element frequency $\omega = (LC)^{-1/2} \equiv 1$.

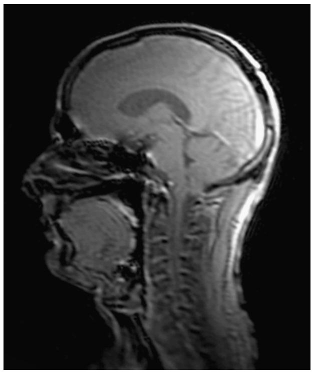

Figure 3 - Sagittal gradient echo head image obtained with the inductively coupled planar array at 3 Tesla. Imaging parameters were TR = 75 ms, TE = 1.7 ms, 5 mm slice thickness. The total scan time for 21 slices was 30 seconds.

Here Ω and Γ are the integral mode numbers and ultimately determine the homogeneity of the magnetic field produced by each of the eigenfunctions. In the weak coupling limit κ and α approach zero, and each element operates independently at the frequency $\omega = (LC)^{-1/2}$ as expected. In the strong coupling limit α and κ are non-negligible and the individual circuit elements combine to produce normal modes of oscillation. In addition, the circuit as a whole exhibits a high-pass behavior in that the lowest mode numbers corresponding to the most homogeneous eigenfunctions appear at the highest frequencies. The dispersion relation for the 5x5 resonator of Fig. 1 is shown in Fig. 2. A total of 25 resonances exist, including ten doublets and five singlets. The general case of a resonator with NxN elements will yield N^2 resonances with $N(N-1)/2$ doublets and N singlets. The *(1,2)(2,1)* doublet, appearing at the next-to-highest frequency, is of particular interest for this work since it can be utilized for quadrature operation and has a B_1 profile suitable for imaging.

Methods and Results: To test the above hypotheses, 5x5 mesh inductively coupled planar and head resonators were constructed for operation at 3 Tesla. For the planar array, each element was tuned to a resonant frequency of 117.0 MHz and had dimensions of 8x8 cm with C = 40 pF. Adhesive backed copper tape was used to form the elements on 1 mm thick Teflon substrates. The experimental eigenvalues are compared with the theoretical values in Fig. 2, and an image is presented in Fig. 3. Outside of the five-fold degeneracy at the single element resonance frequency, all modes were identified. A second array was then constructed on a hemispherical substrate 22 cm in diameter and 13 cm deep, and images of the superior aspect of the human brain as shown in Fig. 4 were obtained from the *(1,2)(2,1)* doublet operating in a quadrature transmit/receive mode.

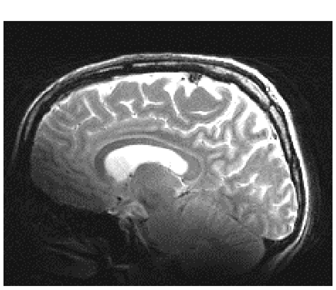

Figure 4 - Sagittal fast spin echo brain image obtained in quadrature with the inductively coupled head array at 3 Tesla. Imaging parameters were TR = 2000 ms, TE = 69 ms, THK=3 mm, and ETL = 8. The total scan time was 4:20.

Conclusion: An interesting point regarding the head array was that the *(1,2)(2,1)* doublet in the eigenmode limit occurred at a frequency more than 15% higher than the single element resonance, and was a strongly varying function of the coupling constant κ. In addition, it is in principle straightforward to increase the operating frequency of this design by decreasing the size of the individual elements. These properties may therefore be useful for high-field applications. Finally, since the zero coupling limit is rarely achieved in practice, the above theory should provide insight into operation of large phased arrays in general.

References: 1. Ballon, D. and Meyer K.L. Two-Dimensional Ladder Network Resonators. *Journal of Magnetic Resonance, Series A*, 111, 23-28, (1994).