Examining the Whiteness of fMRI Noise

O. Friman[1], I. Morocz[1], C-F. Westin[1]

[1]Department of Radiology, Brigham and Women's Hospital, Harvard Medical School, Boston, MA, United States

Abstract

The temporal autocorrelation in fMRI time series is a source of much concern in the neuroimaging community. The serial dependence between samples stems from deterministic signals, such as scanner drifts and cardiac and respiratory cycles, about which we have very little prior knowledge and therefore may have difficulties modeling. In this short exposition, using over 1,000 fMRI data sets, we investigate and demonstrate how the model of irrelevant deterministic components in fMRI time series influences the residual noise structure. In particular, we show that given an adequate model for the nuisance deterministic signals, the noise in a vast majority of fMRI time series can be considered as white, i.e. temporally uncorrelated.

Introduction & Method

Consider the linear model $y = X\beta + S\theta + \varepsilon$, where y is an observed fMRI time series represented as an $(n \times 1)$ vector; X is the design matrix and β the associated regression weights for the BOLD response; S is an $(n \times q)$ matrix containing q temporal basis functions modeling other irrelevant deterministic components and θ the associated weights, and ε is a residual noise vector. Assuming a significant temporal autocorrelation in the residual noise, the main questions are how the size of an experimental effect is best estimated and how valid statistical inference is carried out. Ideally, the residual noise should be temporally uncorrelated, because then it is possible to estimate the energy in a potential BOLD response component optimally, i.e. find a minimum variance estimate of β, by means of an ordinary least squares approach. If the noise is temporally correlated we can adopt a suitable noise model and apply a whitening operation in order to recover optimality. In this work we are interested in how the choice of S, which models nuisance deterministic components, influences the residual noise characteristic. Indeed, the major part of the temporal autocorrelation in fMRI time series arises from deterministic low frequency drifts. Therefore, it is natural to try to model irrelevant deterministic components with a set of low frequency basis functions, such a truncated set of Discrete Cosine Transform (DCT) basis functions, as done in the popular SPM package. However, there might be a smaller set of basis functions that spans a subspace capturing the deterministic components equally well or even better. Given that we in an fMRI data set have tens of thousands of time series, we can try to estimate such a subspace. In fact, a Maximum Likelihood (ML) estimator can be derived [1]. This ML-estimated nuisance model has been proposed for fMRI analysis by Ardekani et al. [2]. In this work we analyze a large number of fMRI data sets using both the DCT and ML models, and compare the temporal autocorrelation in the residual noise. For each data set, the model order q is determined as the one maximizing the Bayesian Information Criterion (BIC),

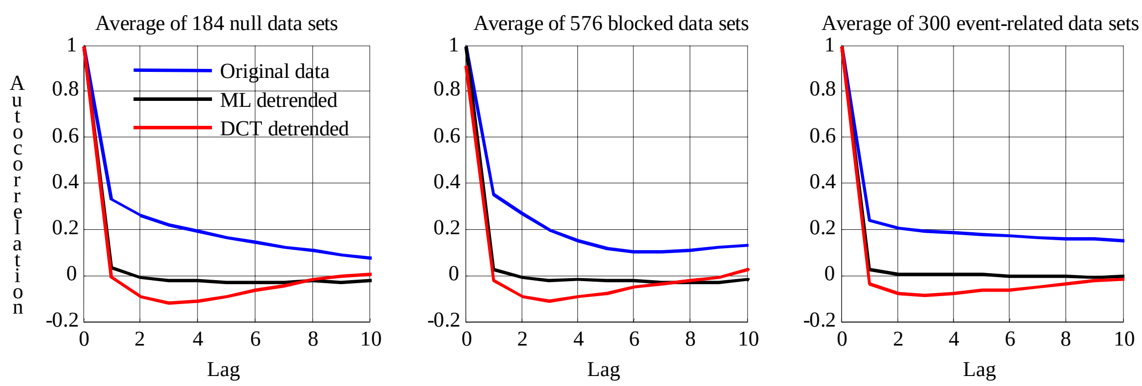
$BIC(q) \approx -n \ln\left(\sigma_q^2\right) - q \ln(n)/2$, where the residual noise variance σ_q^2 is a measure of model fit and n is the length of the time series.

Material

To evaluate the impact of the nuisance model on the residual noise characteristic, 184 rest data sets, 576 blocked design data sets and 300 event-related data sets, comprising about 40 million within-brain time series, were analyzed. Different MR scanners with different field strengths (1.5 T to 4 T), as well as different acquisition sequences (EPI, spiral and 3D PRESTO), were used to acquire these data sets. The length of the time series varies between 85 and 274 samples.

Result & Conclusion

The optimal model order q, i.e. the number of temporal basis functions, was mainly distributed between 5 and 15. Generally, a longer experiment required a greater q. The plots below show the residual temporal autocorrelation for different lags. The curves represent averages for all the time series in all the data sets for each design type. Note that there is a considerable autocorrelation in the original data (i.e. residual noise when no nuisance model is used), clearly indicating the presence of unmodeled deterministic components. The DCT model accounts for most of the low-frequency drifts, consequently reducing the amount of residual noise autocorrelation dramatically (red curve). Thereby, it facilitates the modeling of the noise, which in turn leads to improved estimation of the regression coefficients. The ML-estimated model seemingly does an even better job capturing the deterministic components (black curve), leaving a near-white residual noise structure. The important conclusion we can draw from this result is that it is possible to find a *compact* nuisance model such that the residual noise becomes temporally uncorrelated for a vast majority of the fMRI time series. This assertion is substantiated by the large amount of fMRI data underlying the presented results.



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References

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