## Iterative Reconstruction Methods for Parallel Acquisitions with Non-Cartesian Trajectories

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### INTRODUCTION

Parallel imaging techniques such as SENSE have been shown to enhance MR imaging speed without sacrificing image quality [1,2]. The combination of parallel imaging and non-Cartesian sampling strategies such as spirals creates considerable challenges for image reconstruction. An iterative reconstruction method using gridding [3] has been applied with SENSE for spiral imaging [2]. However, this method bears the same problem as gridding such as time-consuming density compensation computation and image artifacts. In this work, we implemented and evaluated two new approaches for parallel MR image reconstruction of non-Cartesian acquisitions with reduced computation load and improved image quality.

# **METHODS**

For arbitrary k-space samples, the image x and acquired k-space samples b can be related by an encoding matrix E as Ex=b. Iterative reconstruction can be performed by solving the equation  $E^{H}Ex=E^{H}b$ , (where <sup>H</sup> denotes the conjugate transpose) using conjugate gradient method (CGM). For parallel imaging with an array of receiver coils, E is a combination of the encoding matrix of all coils. Standard gridding defines E by convolution, resampling and FFT.

*NUFFT-based method*: Here E was defined using the concept of non-uniform fast Fourier transformation (NUFFT) [4], which can efficiently compute the Fourier transformation for arbitrary *k*-space trajectories. The NUFFT algorithm contains the regular FFT operation F on a grid expanded by a factor of 2 and an interpolation T of grid data onto the *k*-space trajectory. For a parallel acquisition, the definition of E is  $E_{NUFFT}=TF(\Sigma C_i)$ , where  $C_i$  is the coil sensitivity map of coil *i*.

*URS-based method:* The next algorithm was based on the method of uniform resampling (URS) [5]. URS is a reconstruction method used to interpolate non-uniform k-space data **b** onto Cartesian grid points **v** by solving the equation Av=b, where **A** is an interpolation coefficient matrix. The image is obtained by performing an inverse FFT on **v**. Due to the typically large dimension of **A**, it is broken into many small blocks and uses singular value decomposition (SVD) to solve the smaller problems locally. In this formulation, **A** is treated as a whole, but uses only a small number of interpolation coefficients from nearby neighboring points; thus **A** can be stored and computed efficiently in a sparse matrix format while still using the global information in reconstruction. The encoding matrix can be expressed as  $E_{URS}=FA(\Sigma C_i)$ , where **F** is the Fourier transformation.

Essentially, the two methods differed in the order of interpolation and FFT. Since the k-space trajectory and Cartesian grid are the same for all coils, the matrices T and A needed to be computed only once. All three reconstruction methods (gridding, URS, and NUFFT) were implemented in MATLAB (Mathworks, Natick, MA). For gridding and URS reconstruction, Kaiser-Bessel windows with width 4 along  $k_x$  and  $k_y$ , and width 2 along  $k_z$  were applied. Twofold oversampled grids were used in the intermediate steps to reduce the aliasing artifacts. NUFFT reconstruction used a minmax interpolation kernel as outlined in Ref. [4]. Direct manipulation of the  $E^H$  matrix at each step in the CGM was replaced with an inner CGM optimization in order to further minimize aliasing artifacts.

Images were acquired on a GE 1.5 T LX scanner using a four-element phased array coil. For 2D images, the full acquisition used a spiral acquisition with 32 interleaves and 8 ms readout time. The  $256 \times 256$  images covered a field of view of 280 mm for in-plane resolution of 1.1 mm. A reduction factor of 2 was used for SENSE yielding 16 acquired interleaves. For the 3D images, a stack of spirals acquisition with a full acquisition of 64 total spiral interleaves (8 interleaves at each of 8 phase encode steps) covered a  $28 \times 28 \times 2.7$  cm field of view with resolution of  $160 \times 160 \times 8$ . Again a reduction factor of 2 was applied, using 4 interleaves on each of the 8 phase encoding steps.

### **RESULTS AND DISCUSSION**

The 2D reconstruction results are summarized in Fig. 1. 20 iterations were used for all of the three methods. In 3D image reconstruction, illustrated in Fig. 2, 40 iterations were used. The inner CGM iteration was set to 3 for URS and 5 for NUFFT based method. The number of spirals was cut in half for both 2D and 3D samples, i.e. reduction factor R=2. As can be seen, these methods successfully recovered the images from sensitivity and aliasing distortion. Reconstruction using a larger reduction factor is possible, though the total number of iterations may need to be increased. In URS and NUFFT based methods, no convolutions or density compensations are needed. The majority of the operations are multiplication of a sparse



Figure 1. Reference and reconstructed 2D images from a watermelon phantom. (a). Body coil reference. (b) NUFFT reconstruction. (c) URS reconstruction. (d) gridding reconstruction.



Figure 2. Reference and reconstructed 3D images of the same phantom. These are the 4th slice of a  $160 \times 160 \times 8$  image. (a). Body coil reference. (b) Reconstruction from one coil. (c) Sensitivity map for the coil in (b). (d) NUFFT reconstruction. (e) URS reconstruction. (f) gridding reconstruction.

interpolation coefficient matrix and a vector. The coefficient matrices can be precomputed and stored for repeated use with the same sampling trajectory. In this way the reconstruction speed can also be enhanced.

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