k-t BLAST reconstruction from arbitrary *k-t* space sampling: application to dynamic radial imaging

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INTRODUCTION The *k-t* BLAST method (1) describes a general way of removing aliasing in the spatial, temporal frequency domain (*x-f* space) caused by undersampling of the corresponding *k-t* space. The recovery of the unaliased *x-f* signal is facilitated by the fact that dynamic data sets of naturally occurring objects are correlated in space and time, i.e. the data have a very sparse representation in *x-f* space, which consequently has large, empty, and essentially wasteful regions. The aliasing can be removed from *x-f* space, if prior information about the signal distribution in *x-f* space is available, such as from a low resolution training stage of the acquisition. The reconstruction is an inversion problem, which, in the general case, is difficult to solve with direct methods, but it is greatly simplified, if the sampling in *k-t* space conforms to a lattice, which results in simple (lattice) aliasing patterns in *x-f* space. The application of *k-t* BLAST has, therefore, focused mainly on such lattice sampling patterns so far. There are, however, a number of relevant and useful applications, where lattice sampling is not possible, e.g. radial and spiral acquisitions or retrospectively triggered cardiac examinations. This work presents a method for solving the general *k-t* BLAST inversion problem using the iterative conjugate gradient (CG) method and ideas previously introduced for SENSE reconstruction from arbitrary *k*-space trajectories (2). This new method is demonstrated for dynamic radial imaging. Examples from *in vivo* real-time cardiac images are shown.

THEORY, MATERIALS, AND METHODS The general *k-t* BLAST reconstruction equation, which recovers the unaliased *x-f* signal $\boldsymbol{\rho}_{x,f}$ from the *k-t* space samples $\mathbf{m}_{k,t}$, is given by (1):

$$\boldsymbol{\rho}_{x,f} = \boldsymbol{\Theta}_{x,f}\mathbf{E}^H(\mathbf{E}\boldsymbol{\Theta}_{x,f}\mathbf{E}^H + \boldsymbol{\Psi}_{k,t})^+\mathbf{m}_{k,t} = (\mathbf{E}^H\boldsymbol{\Psi}_{k,t}^{-1}\mathbf{E} + \boldsymbol{\Theta}_{x,f}^{-1})^+\mathbf{E}^H\boldsymbol{\Psi}_{k,t}^{-1}\mathbf{m}_{k,t}$$

where $\boldsymbol{\Theta}_{x,f}$ is the signal covariance matrix, $\boldsymbol{\Psi}_{k,t}$ is the noise covariance matrix, and $\mathbf{E}$ is the encoding matrix, which describes the mapping of the fully sampled *x-f* space onto the undersampled *k-t* signal vector. More specifically, the encoding matrix includes the Fourier transform along all dimensions (*x-f* to *k-t*) and the sampling (usually subsampling) on an arbitrary *k-t* trajectory. Superscript *H* denotes the complex conjugate transpose and superscript + denotes the Moore-Penrose pseudoinverse. Generally, the signal covariance matrix is not known and is replaced by a diagonal matrix containing an estimate of the squared *x-f* signal intensities as obtained from training data. Moreover, the noise in *k-t* space is assumed to be uncorrelated and $\boldsymbol{\Psi}_{k,t}$ is replaced with the noise variance. The two forms of the reconstruction equation are equivalent, but derived from the underdetermined and overdetermined cases, respectively. Here, we aim at solving the inversion part of the problem using the CG method, and since the convergence rate of this method is directly related to the conditioning number of the system matrix, we have chosen to solve the latter equation, because it lends itself well to preconditioning with a simple diagonal matrix. Each iteration of the CG method forms the product of the system matrix and a search vector, and this can be replaced with FFT and gridding operations as described in Ref. (2), e.g. the multiplication with $\mathbf{E}$ is replaced by 1) deapodization, 2) FFT to *k-t* space, 3) convolution with gridding kernel, 4) sampling on the *k-t* trajectory.

The proposed method was applied to simulated and *in vivo* data acquired with radial *k*-space sampling. The simulated data consisted of a stationary cylinder with a contracting sphere inside (chest wall and heart). The matrix size was 128x128, 17 projections (12-fold undersampling) were acquired for each time frame with a linearly rotating sampling pattern. The *in vivo* data set was acquired with an SSFP real-time sequence on a Philips Achieva 1.5T whole-body system (Philips Medical Systems, Best, Netherlands); matrix size was 128x128, and 16 projections were acquired each time frame (13-fold undersampling). The prior information needed in the reconstruction was derived from the undersampled data sets by applying a circular Hamming shaped shutter to the *k*-space with a radius that ensured that the Nyquist criterion was not violated in the training data sets. This approach negates the need for a separate training stage in the acquisition. The *in vivo* data set was acquired with a 5-element cardiac coil, and each coil was reconstructed separately with the CG method and subsequently combined using root-mean-square coil combination.

RESULTS The initial tests showed that iterative CG procedure generally converged within 10-20 iterations. Figure 1 shows the simulation results. The upper row shows a fully sampled phantom for reference, the middle row shows the reconstruction from the 12-fold undersampled data, and the bottom row shows the *k-t* BLAST reconstruction results. It is seen that the general *k-t* BLAST reconstruction effectively removes the streaking artifacts. This is also observed in the *in vivo* case. Figure 2 (left) shows a systolic phase from the dynamic series. Undersampling artifacts dominate the reconstruction from the undersampled data, whereas the *k-t* BLAST reconstruction has very few artifacts. A sliding window reconstruction is shown for comparison, which was unable to visualize the contraction of the heart properly because of temporal blurring. This is also illustrated in Fig. 2 (right) where *x-t* plots, from a line passing through the heart, are shown.

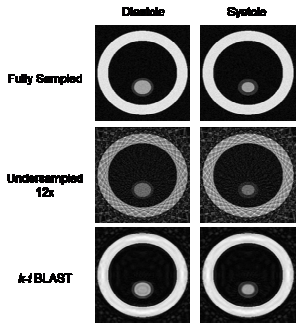

Figure 1. Simulation results.

CONCLUSION *k-t* BLAST reconstruction from non-lattice sampling patterns is possible using an iterative CG method. The *k-t* BLAST method effectively suppresses aliasing artifacts while preserving the temporal bandwidth. The presented approach holds considerable promise not only for arbitrary *k*-space trajectories, but also for applications, where regular sampling of the temporal dimension is not possible, e.g. retrospective cardiac triggering.

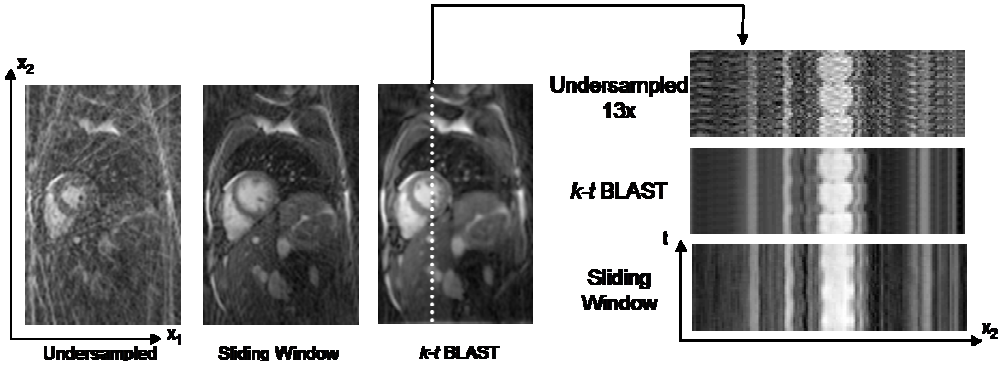

Figure 2. *In vivo* results. Systolic frames on the left, *x-t* plots on the right.

REFERENCES 1. Tsao J et al. Magn Reson Med. 2003 Nov;50(5):1031-42. 2. Pruessmann et al. Magn Reson Med. 2001 Oct;46(4):638-51.