Mutual Information Based NonRigid Image Registration: A Stochastic Formulation

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Abstract Fluid-flow (FF) deformation algorithms have previously used a cost function that minimized differences in pixel intensity between images. An algorithm with such a cost function, however, can be applied only to images acquired from the same modality. To overcome this restriction, we propose a stochastic image deformation algorithm driven by a cost function that maximizes mutual information (MI) across images.

Introduction An image deformation algorithm based on fluid dynamics was first proposed by [1]. In this algorithm, the images to be aligned were modeled as a viscous fluid; deformations were mathematically calculated with the Navier–Stokes partial differential equation (PDE) for fluid dynamics. The body force used to calculate deformations minimized differences in intensity between the subject (to be deformed) and reference (fixed) images. This algorithm, however, required that the registered images be of the same modality, obviously restricting applicability. [2,3] formulates an MI-based algorithm in which the body force is derived from a joint histogram of the subject and reference images smoothed by the Parzen Window method.

Method Our algorithm bypasses the intermediate step of a joint histogram: the body force is derived directly as a function of local image gradients, and then the various probability density functions and the entropy terms are estimated using a method based on Parzen Windows [4]. This stochastic formulation allows our algorithm to escape some local minima. Our algorithm successfully registers images with inverted pixel intensities, as well as images acquired from different modalities. In [1], the authors gave the Navier–Stokes partial differential equation (PDE) governing image deformation as:

$$\mu\nabla^2 v + (\lambda+\mu)\nabla(\nabla\cdot v) + b(u) = 0, \tag{1}$$

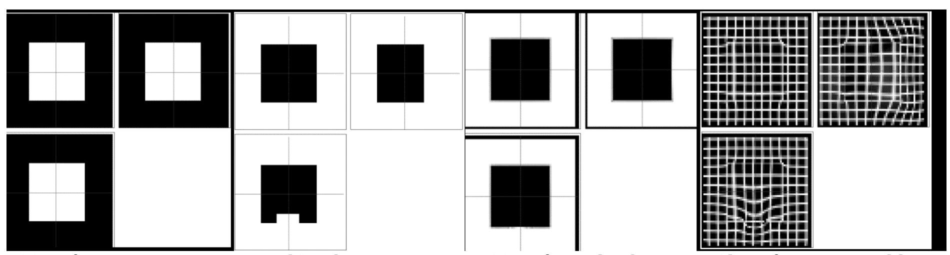
where $\nabla^2 = \nabla^T\nabla$ is the Laplacian operator, $(\nabla\cdot v)$ is the divergence operator, μ and λ are the viscosity constants, *b(u)* is the body force acting on the voxel at location u such that the pixel intensity differences across the images were minimized, and $v(x,t)$ is the velocity of the particle at time t and position x in the Eulerian reference frame. In our algorithm, the body force b(u) was estimated to increase MI across the reference and deformed subject images:

$$b_j = \sum_{i=1}^{N}\left(\frac{W_{\mathbf{t}}(i,j)}{\sigma_{\mathbf{t}}^2} - \frac{W_{\mathbf{s,t}}(i,j)}{\sigma_{\mathbf{s}}^2\sigma_{\mathbf{t}}^2}\right)(t_j - t_i)\,(\nabla T_j - R_{ij}^T\nabla T_i), \tag{2}$$

where R_{ij} be the rotation matrix which rotates the deformation direction vector d_i into d_j, ∇T_j is the image pixel intensity gradient of the image being deformed,

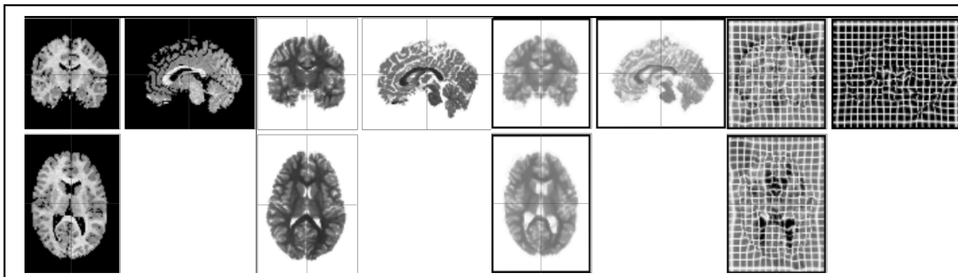
and $W_{\mathbf{s,t}}(i,j) = \left[\dfrac{\exp\left[\frac{-1}{2}\left[\left(\frac{s_j(h)-s_i(h)}{\sigma_{\mathbf{s}}}\right)^2 + \left(\frac{t_j(h)-t_i(h)}{\sigma_{\mathbf{t}}}\right)^2\right]\right]}{\sum_{k=1}^{N}\exp\left[\frac{-1}{2}\left[\left(\frac{s_j(h)-s_k(h)}{\sigma_{\mathbf{s}}}\right)^2 + \left(\frac{t_j(h)-t_k(h)}{\sigma_{\mathbf{t}}}\right)^2\right]\right]}\right]$, $W_{\mathbf{t}}(i,j) = \left[\dfrac{\exp\left[\frac{-1}{2}\left[\left(\frac{t_j(h)-t_i(h)}{\sigma_{\mathbf{t}}}\right)^2\right]\right]}{\sum_{k=1}^{N}\exp\left[\frac{-1}{2}\left[\left(\frac{t_j(h)-t_k(h)}{\sigma_{\mathbf{t}}}\right)^2\right]\right]}\right]$, and s_i and t_i are N, randomly-sampled points from the reference and subject images, respectively.

Results Fig. 1 (a) and (b) show a set of synthetic reference and subject 3D images. The reference is a binary representation of a cube at the center of the image. The subject image is identical to the reference image except that it has an indentation on the bottom side, and with inverted pixel intensities. Three orthogonal sections of the images are shown. Fig. 1 (c) shows the final deformed subject image; Fig. 1 (d) shows the final estimated deformation field. While the pixel intensities of the corresponding regions are inverted in the reference and the subject images, the estimated deformation field is smooth and feasible. Fig. 2 (a) shows a real 3D MR image used as the reference image; Fig. 2 (b) shows a different 3D MR image used as the subject image with pixel intensities inverted. Fig. 2 (c) shows the final deformed subject image; Fig. 2 (d) shows the estimated deformation field.



(a) Reference (b) Subject (c) Deformed Subject (d) Deformation Field

Fig. 1. Synthetic Image with inverted pixel intensities.



(a) Reference (b) Subject (c) Deformed Subject (d) Deformation Field

Fig. 2 Matching different real subject MR Images with inverted pixel intensities

Discussion Our algorithm for FF deformation of images maximizes MI between images. We calculated the body force based on variations in MI for small changes in the deformation field. This body force is a function of the local image gradient directly derived from the images to be matched. Synthetic and real-subject trials verify that our algorithm reliably deforms images of different modalities; it also successfully deforms images when pixel intensities are inverted between the subject and the reference images.

References

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