Iterative Phase Correction of Multi-Shot DWI Using Conjugate Gradient Method

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INTRODUCTION: Multi-shot sequences are useful for high-resolution diffusion-weighted imaging (DWI) (1,2,3). A common problem encountered in multi-shot image acquisition is non-linear phase variation from shot to shot (4). This phase error is usually caused by patient motion during diffusion encoding periods. Typically this error is corrected for by subtracting a low resolution phase map from each shot. This phase map can be obtained either from an extra navigate image or from the **k**-space data of a self-navigated trajectory (e.g. variable density spirals). This simple algorithm can effectively remove the phase error if image reconstruction can be separated from phase correction. However, in many cases, these two processes can not be separated; thus can not be performed sequentially. For example, when each shot undersamples the **k**-space, effect of aliasing causes the phase error at one location appearing at other locations. The resulting non-localized phase error can no longer be corrected through a simple phase subtraction. Here, we present a method that performs phase correction simultaneously with image reconstruction. This novel method combines the conjugate gradient (CG) method with a least-square estimation of the image to be reconstructed. Successful phase correction is demonstrated for multi-shot DWI with self-navigated interleaved spirals (SNAILS) (3).

METHOD: Motion during diffusion encoding periods introduces phase errors that cause serious image degradation in multi-shot DWI. This extra phase can be incorporated into the image encoding function if treated as an additive perturbation term (4). Written in a matrix format, data acquired with a multi-shot sequence can be expressed as,

$$\mathbf{d} = \mathbf{E}\,\mathbf{m} \qquad [1]$$

Here, **d** is the **k**-space data stored in a column vector; **m** is the image space data stored in the same fashion; and **E** is the encoding matrix. The size of **E** is N_k x N^2 , where N_k the total number of **k**-space sampling points and N is the image size. For the *n*-th shot, the elements of matrix **E** are

$$\mathbf{E}_{(k,n),\rho} = \exp(-i\,\mathbf{k}_{k,n}\mathbf{r}_\rho)\,p_n(\mathbf{r}_\rho) \qquad [2]$$

Here, $\mathbf{k}_{k,n}$ is the *k*-th sampling point of the *n*-th shot, and $\mathbf{r}_\rho$ is the ρ-th pixel of an image.

Two cases need to be considered in order to solve Eq. [1]. First, if each shot samples **k**-space at or above the Nyquist rate (e.g. PROPELLER), then $p_n(\mathbf{r}_\rho)m(\mathbf{r}_\rho)$ can be easily computed for each shot separately using inverse Fourier transform. To take advantage of the fast Fourier transform (FFT) algorithm, a gridding procedure may be necessary if the sampling trajectory is non-Cartesian. Following an inverse FFT, the phase error $p_n(\mathbf{r}_\rho)$ can be simply removed through a complex-number division.

Second, if each shot samples the k-space below the Nyquist rate (e.g. SNAILS), then $p_n(\mathbf{r}_\rho)m(\mathbf{r}_\rho)$ cannot be computed from each shot without suffering aliasing artifacts. Consequently, the matrix **E** that contains all the phase information has to be inverted. This inversion can be effectively performed using a least-square estimation:

$$\mathbf{m} = (\mathbf{E}^H\mathbf{E})^{-1}\mathbf{E}^H\,\mathbf{d} \qquad [3]$$

Typically E is a very large matrix. As a result, it is numerically impractical to estimate the image through direct matrix inversion and multiplication. However, this equation can be solved iteratively. For the efficiency of numerical evaluation, Eq. [3] can be rewritten as,

$$(\mathbf{E}^H\mathbf{E})\,\mathbf{m} = \mathbf{E}^H\,\mathbf{d} \qquad [4]$$

Various techniques have been developed to solve large systems of linear equations. Here, we apply the conjugate-gradient (CG) method by exploring the similarity between Eq [4] and the equation of sensitivity encoding (SENSE) (5). We observe that Eq.[4] is equivalent to the SENSE reconstruction problem when $p_n(\mathbf{r}_\rho)$ is treated as a pseudo "coil sensitivity" that varies from shot to shot.

Therefore, in the second case, to reconstruct an image from undersampled multi-shot diffusion data, we first estimate a low resolution phase map $p_n(\mathbf{r}_\rho)$ for each shot. The right hand side of Eq.[4] is initially computed by directly removing the phase from each shot prior to summing all the data. During each iteration of CG, $(\mathbf{E}^H\mathbf{E})\mathbf{m}$ is calculated by using a combination of forward and backward gridding when the **k**-space trajectory is non-Cartesian (Figure 1). Without loss of generality, other speed-up techniques such as the convolution-based SENSE can also be applied to calculate $(\mathbf{E}^H\mathbf{E})\mathbf{m}$.

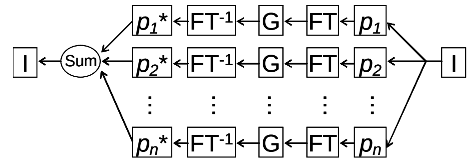

Fig 1 – algorithm flow chart to be incorporated into the CG iteration. p_n: phase error of *n*-th shot. **G**: gridding and inverse gridding. **I**: intensity correction.

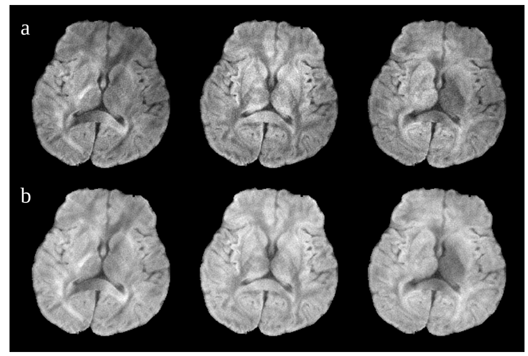

Fig 2 – (a) initial and (b) final images for 3 diffusion directions.

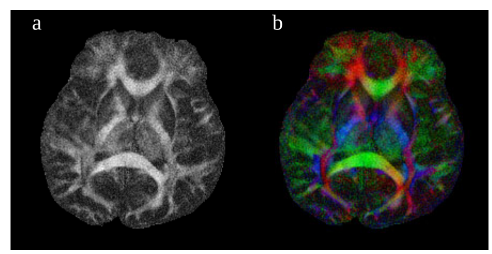

Fig 3 – (a) FA; (b) color FA. Red: A/P; green: L/R; blue: S/I.

In vivo diffusion-weighted images were acquired using SNAILS (3) on a GE Signa 1.5T whole-body system. The parameters were: number of interleaf = 28, FOV = 22cm, TR = 2.5s, TE = 67ms, and b = 800s/mm^2. Six diffusion-encoding directions were applied to measure the diffusion tensor. Low resolution phase maps were estimated using the center k-space data from each interleaf.

RESULTS: Figure 2 shows estimated initial images and phase-corrected final images after 5 iterations. The initial images suffer from various degrees of signal cancellation because the motion-induced phase can not be completely subtracted from each interleaf. The signal loss is clearly restored in the corrected final images, resulting in a much higher signal to noise (SNR) ratio. Figure 3 shows the computed fraction anisotropy (FA) map and color coded FA map.

DISCUSSION: We have established a mathematical framework for simultaneous image reconstruction and phase correction for multi-shot DWI. The image is computed iteratively using the CG method. In vivo experiments demonstrate that the signal cancellation caused by phase error can be successfully recovered using this algorithm (Fig 2). Due to the enhanced SNR, the number of signal averages required for high resolution DTI can be reduced, thus improving the acquisition speed.

The problem of phase correction for multi-shot DWI was previously treated by Karla et al. (4). In their approach, Eq.[3] was simplified by assuming that $(\mathbf{E}^H\mathbf{E})^{-1}$ was an identity matrix, which was equivalent to treating the data as fully sampled. The method proposed in this abstract not only corrects for phase error of fully sampled data (e.g. PROPELLER) but also corrects for undersampled data (e.g. SNAILS). If each shot samples **k**-space at or above the Nyquist rate, this method reduces to the straight forward phase subtraction method. Such kind of sampling strategy and phase correction scheme are seen in PROPELLER DWI. However, if each shot undersamples **k**-space, the simple phase subtraction method cannot completely remove the phase error. In this case, Eq.[3] needs to be solved.

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