A Spherical Harmonic Approach to Q-Ball Imaging

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INTRODUCTION: Q-ball imaging (QBI) has been introduced as a technique for high angular resolution diffusion imaging (HARDI) that is especially useful when the tensor model is inadequate, as in the case of intersecting white matter fiber populations [1]. Among several methods that have been proposed, QBI has the advantages of model independence, relatively easy implementation, and direct relationship to the diffusion spectrum space. While a number of empirically satisfactory results using QBI have been described in the literature, the performance of the method in terms of angular resolution, artifacts and sensitivity to noise has yet to be fully elucidated. We propose a reformulation of QBI in terms of spherical harmonics that facilitates analysis of QBI and leads to a more computationally efficient algorithm for reconstructing orientation distribution functions (ODFs) from HARDI data distributed arbitrarily on the unit sphere. Computer simulations and *in vivo* experimental results are provided to verify the theoretical results.

METHODS: Angular dependence of spin displacement in QBI is estimated in the form of an orientation distribution function, $h(\theta,\phi)$, where θ (azimuth) and ϕ (elevation) define points on the unit sphere in spherical coordinates. According to the Funk transform, $h(\theta,\phi)$ is constructed from diffusion measurements $E(\theta,\phi)$ by path integration along great circles [2]. In the proposed method, the ODF is expressed in terms of its order-L spherical harmonic basis,

$$h(\theta,\phi)=\sum_{l=0}^{L}\sum_{m\le|l|} f_m^l Y_m^l(\theta,\phi), \qquad (1)$$

where f_m^l represent the harmonic series coefficients and $Y_m^l(\theta,\phi)$ denote the spherical harmonic basis functions of order l and degree m [3]. This parameterization allows great circle integration to be transformed into simple scalar multiplication, such that the harmonic coefficients of $h(\theta,\phi)$ can be obtained as $f_m^l = 2\pi P_l(0) g_m^l$, where $P_l(x)$ is a Legendre polynomial and g_m^l are the harmonic series coefficients of $E(\theta,\phi)$. From an implementation standpoint, the ODF is generated in three steps: (1) calculation of g_m^l by harmonic decomposition of $E(\theta,\phi)$, (2) computation of f_m^l from g_m^l by multiplication with $P_l(0)$, and (3) harmonic synthesis according to equation (1). Because spherical harmonic synthesis and decomposition take the form of matrix multiplication, reconstruction is computationally efficient and requires only linear operations. The matrices are computed only once based on the diffusion gradient orientations. Unlike the original implementation of QBI, no interpolation is necessary.

Simulations and actual HARDI experiments were performed to demonstrate the validity of the spherical harmonic Q-ball approach. To simulate crossing fiber populations, data were generated according to a prolate tensor model with added Gaussian noise [4]. For *in vivo* performance analysis, HARDI was performed on five normal adult volunteers on an EXCITE 3T scanner (General Electric, Milwaukee, WI). Parallel acquisition with an 8-channel head coil and a single-shot echoplanar spin-echo pulse sequence allowed whole brain imaging with 2 mm isotropic resolution in 45 minutes (TR/TE = 18s/84ms, b=3000s/mm^2, NEX=1, SENSE factor 2). Diffusion gradients were prescribed along 131 independent directions uniformly distributed over the surface of a sphere via electrostatic repulsion [4]. Processing was done using software written in MATLAB (Mathworks, Natick, MA).

RESULTS & DISCUSSION: The true ODF in a HARDI experiment is a function of the underlying white matter architecture, the voxel size, and the b value. Spherical harmonic analysis, equivalent to Fourier transformation on the sphere, allows quantitative evaluation of angular resolution and noise properties for QBI. With the proposed approach, the reconstructed ODF is related to the true ODF by convolution with a spherical point spread function (PSF), shown in Fig. 1(a) for model order L = 2, 4, 6 and 8. The ability to separate crossing fibers depends upon the width of the PSF, which is inversely proportional to model order (as in the cartesian Fourier series). Also analogous to traditional Fourier analysis, higher-order harmonics have lower SNR, and the SNR of the reconstruction falls with increasing model order (for a fixed number of measurements). Unlike a standard Fourier series, noise propagation from the measurement domain also depends upon the relative conditioning of the harmonic decomposition matrix.

The effects of model order on angular resolution and noise are plotted in Fig. 1(b) for two simulated fiber tracts with fractional anisotropy 0.7 and b=3000 s/mm^2. The blue (solid) curve, which is inversely proportional to L, depicts the minimum intersection angle that can be resolved as a function of harmonic model order. The green (dashed) curve, which is proportional to the square root of L, illustrates the mean square error between the noise-free and reconstructed ODFs when measurements are corrupted by Gaussian noise with mean 0 and variance 1 (averaged over 1000 trials of noise). As demonstrated in Fig. 1(c), tracts crossing at 60° can be separated when L=6 (*top left*) but not when L=4 (*top right*), as predicted by the analysis. For tracts crossing at 90° with SNR=5, the effects of noise are minimal when L=4 (*bottom left*) but prominent when L=8 (*bottom right*), giving rise to artifactual peaks that could be erroneously interpreted as distinct fiber tracts.

Fig. 2 shows experimental results from an adult volunteer. Conventional QBI (a) and the proposed method with L=6 (b) yield qualitatively similar results within a coronal region of interest containing intersecting tracts from the corpus callosum (red), superior longitudinal fasciculus (green) and centrum semiovale (blue). The interpolation step in standard QBI decreases angular resolution, however, such that crossing fibers from the superior longitudinal fasciculus are more clearly resolved with the proposed method.

Of note, the proposed technique unifies QBI with the spherical deconvolution method recently proposed by Tournier *et al* [5], in which the ODF is also reconstructed by weighting the spherical harmonic coefficients of the measured signal attenuation. In contrast to the proposed technique, however, harmonic weights in spherical deconvolution are selected empirically to calculate a "response function." The results are then highly dependent on which voxels are selected, such that numerical stability, noise performance and angular resolution are not readily assessed. Computing the ODF harmonic coefficients in terms of the Legendre polynomials maintains the model independence of QBI and allows for more generic performance analysis.

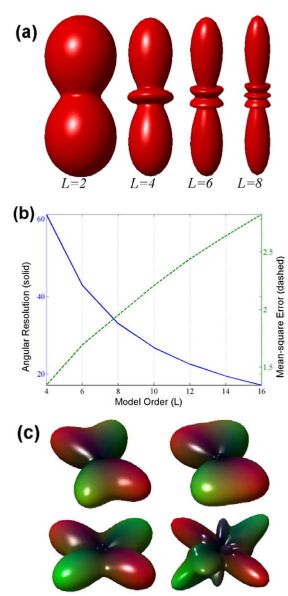

Figure 1

CONCLUSION: A new formulation of Q-ball imaging based on the spherical harmonic representation has been proposed. This technique results in more computationally efficient reconstruction and allows familiar tools from Fourier analysis to be used to predict artifacts, noise performance and angular resolution. This method may improve accuracy in fiber tracking, resilience to partial volume averaging, and characterization of white matter disease in regions of complex cytoarchitecture.

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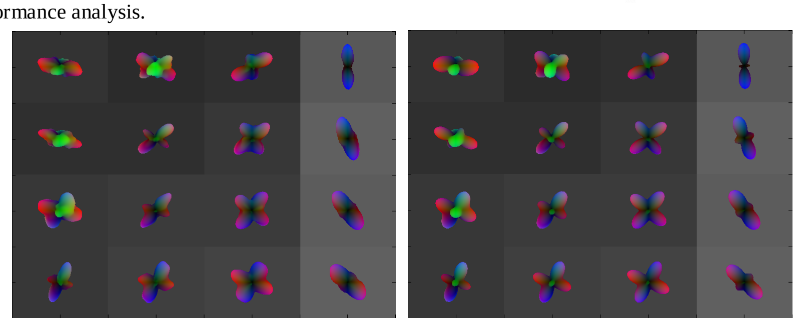

Figure 2 **(a)** **(b)**