Improved characterisation of crossing fibres: optimisation of spherical deconvolution parameters using a minimum entropy principle

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Introduction: Diffusion-weighted MRI can be used to extract the orientation of white matter fibres. Currently, the most common way of estimating the orientation of the fibres makes use of the diffusion tensor model. However, this model is unable to characterise voxels containing multiple fibre orientations [1]. We have recently proposed a technique that is able to estimate the distribution of fibre orientations directly from the diffusion-weighted data, using the concept of spherical deconvolution [2]. The technique requires a *response function* and associated low-pass filter (to mitigate the effects of noise). However, the optimal shape for these functions will in general depend on factors such as the SNR, the number of diffusion-encoding directions used, the *b*-value, and physiological factors such as the level of maturation of the brain. In this study, we propose an objective way of estimating and optimising these functions, using a minimum entropy concept, to ensure that the results provided by the technique are robust and reproducible. In addition, we propose a normalised version of the entropy as a natural alternative measure of anisotropy for non-tensor-derived fibre orientation distributions.

Theory: The spherical deconvolution technique works by assuming a *response function* R(θ), corresponding to the diffusion-weighted signal attenuation that would be measured as a function of orientation for a typical coherently oriented fibre bundle aligned with the *z*-axis. The fibre orientation distribution (FOD), F(θ,φ), is then obtained by deconvolving this response function from S(θ,φ), the diffusion-weighted signal attenuation actually measured during the experiment. Using ⊗ to denote the spherical convolution operation, this can be written:

$$S(\theta,\phi) = R(\theta) \otimes F(\theta,\phi)$$

which can be performed simply and efficiently using the spherical harmonic series:

$$S^l_m = R^l \cdot F^l_m$$

where S^l_m and F^l_m denote the (l,m) spherical harmonic coefficient of S(θ,φ) and F(θ,φ) respectively, and R^l is related to the $(l,m=0)$ spherical harmonic coefficient of R(θ) (all $m\neq0$ coefficients are zero due to the assumed axial symmetry of the response function). The deconvolution operation can be performed by inverting the equation. However, as higher orders are included in the harmonic series, the operation becomes increasingly unstable due to the presence of noise. This can be overcome by introducing filtering, whereby the higher harmonic orders are attenuated by scaling the corresponding coefficients by a factor β^l, at the expense of angular resolution in the resulting FOD. The spherical deconvolution operation can then be written:

$$F^l_n = (\beta^l / R^l) \cdot S^l_m = \rho^l \cdot S^l_m$$

where $\rho^l = (\beta^l / R^l)$ corresponds to the *effective* response function. In our previous studies [2], the true response function was estimated directly from voxels assumed to contain a single fibre population (i.e. those with the highest anisotropy), and empirically determined filter coefficients were applied to yield the effective response function. However, this method of estimating the effective response coefficients is sub-optimal and operator dependent. In this study, we propose a way to objectively determine the optimal effective response coefficients directly from the data, by minimising the entropy in the generated FOD.

The entropy quantifies the amount of information contained in a distribution [4]. If the FOD contains a single well-defined peak, its entropy will be low. If the FOD has a large number of peaks, or contains very broad peaks, its entropy will be high. The entropy therefore provides a measure of the coherence of the fibre orientations in a given voxel. In this study, we produce maps of the *redundancy*, a normalised version the entropy, scaled between zero (maximum entropy) and one (minimum entropy) [4]. We propose this measure as a natural alternative to the anisotropy indices commonly used to characterise the diffusion tensor; it is applicable to any FOD, whether derived using spherical deconvolution or otherwise.

Methods: Diffusion-weighted data were acquired from a healthy 26-year old volunteer on a 1.5T Siemens Vision system using a twice-refocused EPI sequence [5] (b = 2971 s/mm^2, TE = 140 ms, FOV = 384 × 384 mm, matrix size 128 × 128 zero-filled to 256 × 256, slice thickness 3 mm, 40 contiguous slices, 60 directions, 6 b=0 images, 3 repeats). The optimal ρ^l coefficients were estimated by minimising the sum over all voxels of the entropies of their respective FOD, weighted by the mean signal attenuation in each voxel (to remove the CSF contribution). In addition, it was necessary to add an extra constraint to minimise the presence of negative values in the FOD, which are induced by the presence of noise, but are obviously physically impossible. The initial values used to start the algorithm correspond to the unfiltered ideal response function: a flat disc (i.e. as would be measured for an idealised fibre bundle with unit anisotropy).

Results & Discussion: Figure 1 shows maps of the redundancy in the FOD, produced using different effective response coefficients. Map A was produced using the response function measured from the data, with an empirically determined filter [2]; it appears noisy, with relatively poor grey/white matter contrast. This poor contrast would imply that the distribution of orientations in the grey matter exhibits similar levels of coherence than in many white matter structures, which is biologically implausible. On the other hand, map B, produced using the optimised effective response coefficients, has higher overall intensity, improved noise characteristics and higher grey/white matter contrast than map A.

Figure 1 also demonstrates the effect of the effective response on the reconstructed FOD. Although the main orientations in the FODs obtained in (A) are well defined, other small lobes can also be seen that are mainly due to noise. In certain voxels, the amplitude of these lobes can become significant, as can be seen in the third example FOD. In contrast, the FODs in (B), produced using the optimised effective response coefficients, have fewer and smaller noisy side lobes, at the expense of only a small reduction in angular resolution.

Conclusion: We have described a method for estimating the effective response coefficients for use with the spherical deconvolution method that provide the optimal compromise between the angular resolution of the FOD, and the stability of the algorithm with respect to noise. This should further improve the reliability of tractography techniques that make use of the spherical deconvolution technique. We have also proposed an alternative anisotropy measure, the redundancy, suitable for the characterisation of *any* non-tensor-derived FOD.

References: [1] Alexander DC *et al.* MRM 48:331 (2002). [2] Tournier JD *et al.* NeuroImage 23:1176 (2004). [3] Healy DM *et al.* J. Mutlivar. Anal. 67:1 (1998). [4] Mackay DJC *Information Theory, Inference and Learning Algorithms*, Cambridge University Press, page 32 (2003). [5] Reese TG *et al.* MRM 49:177 (2003).

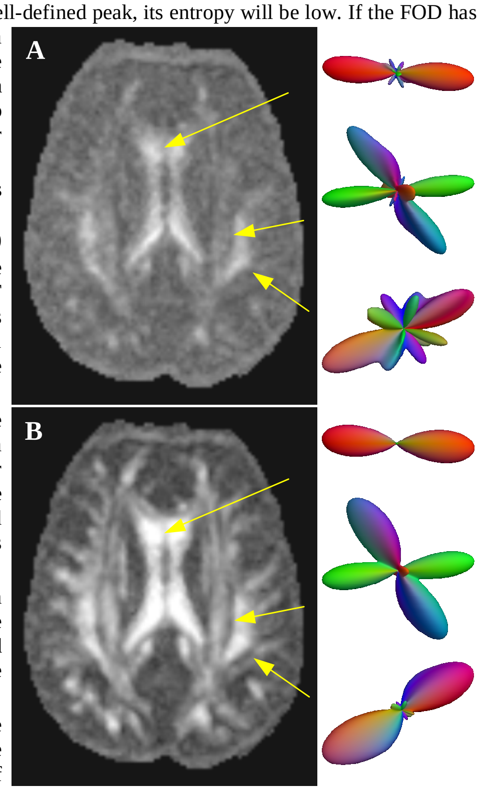

Figure 1: left: maps of the redundancy in the FOD, generated using two different sets of filtered response coefficients. Right, FODs generated from the voxels indicated, using the same effective response coefficients, and coloured according to direction (red: left-right, green: anterior-posterior, blue: inferior-superior). (A) produced using the response function measured from the high anisotropy voxels, with filtering parameters set empirically. (B) produced using the effective response parameters obtained by minimising the entropy of the FOD. Both maps are displayed using identical windowing parameters.