Simulating Helium-3 Diffusion in the Lung: Comparing the "Cylinder-Model" with Simulated 3D Alveolar Ducts

S. Fichele[1], M. N. Paley[1], N. Woodhouse[1], Z. Said[1], P. D. Griffiths[1], E. J. van Beek[1], J. M. Wild[1]

[1]Academic Unit of Radiology, University of Sheffield, Sheffield, United Kingdom

Introduction: Devising a complete analytical theory that describes Helium-3 diffusion in lung tissue is difficult. The most recent attempt makes the approximation that each alveolar duct can be treated as a smooth walled, infinitely long cylinder, for which the diffusion equation can be solved [1]. In this case diffusion is considered anisotropic, and is categorized by two components; one along the principle axis, D_L, and the other transverse, D_T. The results from the transverse component can be used to predict the average radii of the alveolar ducts for which in-vivo experiments have yielded plausible results.

Here finite difference simulations have been used to model Helium-3 diffusion during a typical PSGE [2] experiment used in hyperpolarized gas imaging [3]. We simulated diffusion in 3D alveolar ducts using the model shown in Figure 1. The aim was to investigate how well the "cylinder-model" could estimate the radii of the simulated structures, given that they are not actually cylinders.

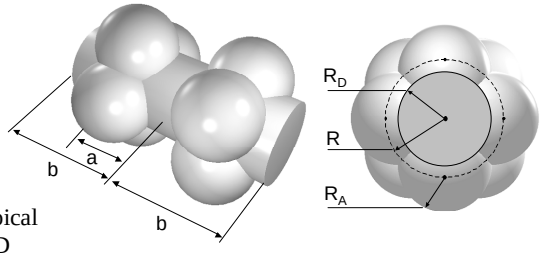

Figure 1: A 3D model for the alveolar duct, generated from a cylinder and 2N spheres. Here N = 4.

Methods: In brief, the magnetization was calculated at each node in a 3D grid using a finite difference method [3,4]. For each time step the phase of the magnetization was incremented according to the gradient strength at that position. A boundary wrapping technique was employed where magnetization at the periphery of the simulation volume could diffuse to the opposite side [3]. This eliminates boundary anomalies and allows diffusion to be investigated in infinitely long alveolar ducts. The 3D alveolar structures were generated from the loci of a cylinder, and 2*N* spheres, see Figure 1. The alveolar spheres were arranged in two groups, one at the "front" and the other at the "back", each consisting of *N* alveoli. The orientation of the two groups about the main axis differed by $\alpha/2$, where $\alpha=360/N$. All software was developed in *C*, and *Matlab*.

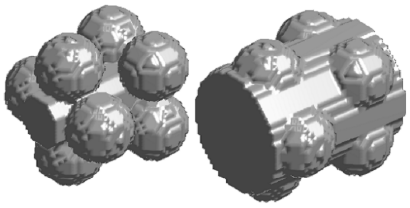

Figure 2: Two example structures. R_A and R are fixed while R_D has been changed.

ADC values were calculated for a range of geometrical parameters, using b-values up to 10 cm^{2}s. Structures were systematically generated with fixed R_A and R while changing R_D from $R-R_A$ to $R+R_A$, e.g. Figure 2. For each individual structure, simulations were conducted for 30 gradient orientations from the principle axis (0, 1/30 π, … 29/30π). The results were then summed according to the "cylinder theory" [1] and the data then fitted to find an estimated value for the alveolar radius, R_{fit}. This value was then compared to the effective radius of the structure, which was calculated as $R_{eff} = \sqrt{\text{Volume}/(\pi L)}$, where L is the length of the structure along the principle axis.

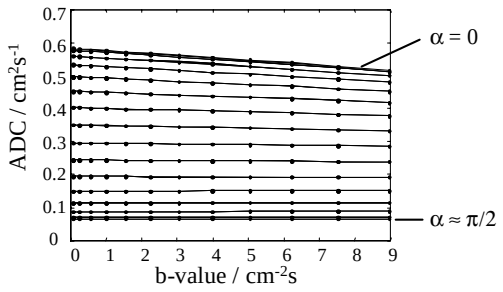

Figure 3: The ADC for one structure taken for gradient angles at α = 0, 1/30π … 29/30π to the main axis. Structure: *R*=195, R_A=134, R_D=225, *a*=300, *b*=600 μm, N=4

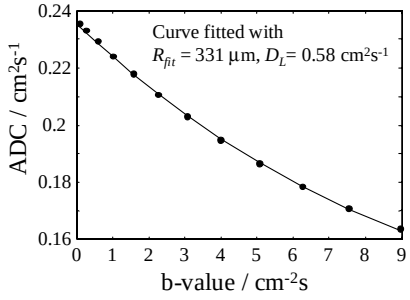

Figure 4: Results (from figure 3) summed according to cylinder model and was compared to theory. R_{eff} = 306 μm, whereas R_{fit} found to be 331 μm.

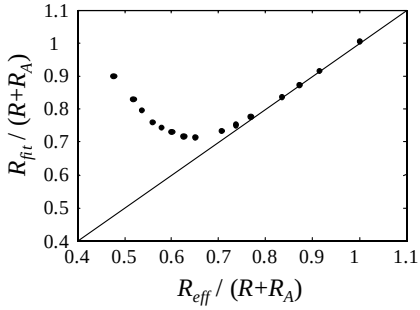

Figure : Comparison of R_{fit} with R_{eff}. Simulated structures: *R*=255, R_A=135 *a*=120, *b*=240 μm, N=5, R_D varied from 120 to 390 μm. The line is plotted for $R_{eff} = R_{fit}$

Results: A typical set of results, for a constant diffusion time (1.8 ms), are shown in Figure 3 for different gradient angles for the same structure. In Figure 4, the results from the same simulation have be summed according to the "cylinder-model", and then fitted to reveal an estimate value for the alveolar duct radius, R_{fit}. The data fits well with the cylinder model trend, however, R_{fit} does not agree with R_{eff}. In Figure 5 the fitted radii for a set of simulations is compared to the effective radii for each simulated structure. The results demonstrate that R_{fit}, doesn't correlate linearly with R_{eff}. for ratios $R_{eff}/(R+R_A)$ below ≈0.8.

Conclusion: Finite difference simulations provide a good way of investigating diffusion in complex structures. We have demonstrated that the "cylinder-model" closely fits the results from 3D simulated alveolar ducts, however, the fitted data tends to overestimate the effective radii. Also, the estimation of radii does not necessarily correlate with the actual effective radius for structures where R_D is changing, i.e. in diseases like emphysema.

References

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