Comparison of the Chirp z-Transform and Interpolation Techniques for Field-of-View Scaling

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Abstract

A version of a chirp z-transform [1] was programmed enabling phase-preserving FOV scaling for data sets with the zero of k-space in the middle. The method is important for all single-point imaging (SPI) sequences [2,3] such as SPRITE when used with multiple data acquisition for T_2^* mapping or signal averaging [4]. The method is most desirable for nuclei with short relaxation times like sodium at high field. Here, the chirp z-transform is compared with a conventional interpolation approach following fast Fourier transformation.

Introduction

In principle, all SPI sequences are capable of producing T_2^* maps by sampling multiple data points following one excitation pulse. These multiple data points are related to different k-spaces with the corresponding evolution time. In the SPRITE sequence, there are constant phase encode gradients on during acquisition, resulting in a FOV change between successive points and therefore k-spaces. The signal per voxel volume therefore scales proportionally. Hence, normalisation of the image intensity is needed such that the signal intensity per voxel volume is preserved. This can be done by an interpolation step following a conventional Fourier transform, or by using the chirp z-transform. The chirp z-transform presented here takes the different scale factors into account correctly and the resulting images are normalised accordingly. The signal intensity is preserved and the T_2^* decay can be measured. We have modified the chirp z-transform such that phase information is preserved and coherent summation of the transformed images can now be used for signal averaging.

Theory

The conventional definition of the chirp z-transform is based on the idea of transforming complex-valued data on equidistant points on an arbitrary arc in the complex plane. The scaling effect of the multiple point acquisition can now be understood as sampling the data along different sections of the unit circle (Eq. 1.1).

$$X(z_k)=\sum_{n=0}^{N-1}x[n]\left(AW^{-k}\right)^{-n}=\sum_{n=0}^{N-1}x[n]A^{-n}W^{nk}=\sum_{n=0}^{N-1}x[n]A_0e^{-i2\pi\theta_0 n}W_0e^{i2\pi\phi_0 nk} \qquad (2.1)$$

By choosing the corresponding phase offsets correctly prior to transformation (Eq. 1.2), the additional phase factor of the conventional chirp z-transform can be omitted completely (Eq. 1.3), thereby preserving the signal intensity, and more importantly, phase information.

$$\begin{aligned} &A_0=W_0=1\\ &A=1,\quad \Theta=0\\ &W=e^{-i\Phi},\quad \Phi=2\pi r/M,\quad r\le 1\\ &z_k=W^{-k}=e^{i\Phi k}\end{aligned} \qquad (1.2)$$

$$X(z_k)=\sum_{n=-N/2}^{N/2-1}x[n]e^{-i\Phi nk}=\sum_{n=-N/2}^{N/2-1}x[n]e^{-i2\pi rnk/M};k=-N/2,\dots,-1,0,\dots,N/2-1 \qquad (1.3)$$

$$X(z_k)=\sum_{n=-N/2}^{N/2-1}x[n]e^{-i\Phi n^2/2}e^{-i\Phi k^2/2}e^{i\Phi(k-n)^2/2};k=-N/2,\dots,-1,0,\dots,N/2-1 \qquad (1.4)$$

It should be noted that in the form (1.3) the chirp z-transform is precisely the adjoint to the discrete version of a scaled Fourier transform [5]. This is an easy access point to incorporate filtering and regulation strategies in the chirp-z transform. Calculating the convolution in Eq. 1.4 by the fast Fourier transform, the floating point operation count (in two dimensions) is $O(N^2 \log N)$ compared to $O(N^3)$ which is needed to perform the interpolation approach.

Methods

A chirp z-transform as described above was coded in C using the netlib fftpack capable of transforming vectors of arbitrary length and compiled with the gnu C-compiler version 2.95 under Linux. A virtual phantom in two dimensions was used to test limitations in the transformation of higher spatial frequencies with increasing scaling factors. The phantom had a circular shape with uniform density, and consists of two centric compartments with different relaxation rates. The resolution was chosen to be 128^2 pixels, and 20 time points with a step size of 50ms were taken to sample the intensity decay. The T_2^* decay was set to 1000ms and 400ms, respectively. The FOV scaling ranged from 1:1 to 1:20. Finally, 5% relative white noise was added separately to the real and imaginary part of the data.

Results and Discussion

Fig. 1 and 3 show the reconstructed profiles using the chirp z-algorithm and a bilinear interpolated Fourier transformation, respectively. Fig. 2 and 4 depict the corresponding decay curves for selected pixel locations starting in the centre of image space and going outwards to the boundary of support.

Chirp z-Transform

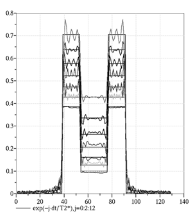

Figure 1: Profiles of selected time points at X=65 computed by the chirp-z transform, and the true signal intensity (straight lines).

Intensity Decay

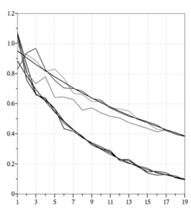

Figure 2: Intensity decays for selected pixels in both compartments computed by the chirp z-transform, and the true signal decay (smooth lines).

Interpolated FT

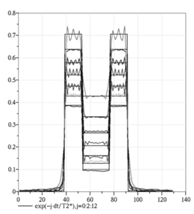

Figure 3: Profiles of selected time points at X=65 computed by bilinear interpolated Fourier transformation, and the true signal intensity (straight lines).

Intensity Decay

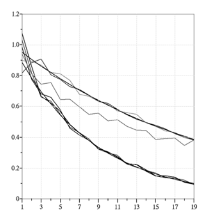

Figure 4: Intensity decays for selected pixels in both compartments computed by bilinear interpolated Fourier transformation, and the true signal decay (smooth lines).

We have shown that the new chirp z-algorithm is capable of FOV scaling with reasonable accuracy up to a factor 1:2. Furthermore, it preserves signal intensity and signal phase. The accuracy is diminished at higher spatial frequencies but can be improved with increased resolution. The results from the new chirp-z are comparable to those from interpolated image data, and in some cases the interpolation method gives less oscillation in the reconstructed images, especially at locations with high intensity jumps. By virtue of its shorter computation time, the chirp z-transformation is more likely to be applied in the three-dimensional case. While the chirp z-method is a fast method, the interpolation approach should not be neglected if resolution is of interest in applications for T_2^* mapping and signal averaging with FOV scaling in sequences such as SPRITE.

References

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