Precalculated Iterative Next Neighbor re-Gridding (PINNG): Accurate and Efficient Reconstruction for Non-uniformly Sampled K-space Data

H. Moriguchi[1], J. L. Duerk[1]

[1]Departments of Radiology and Biomedical Engineering, University Hospitals of Cleveland / Case Western Reserve University, Cleveland, OH, United States

Introduction

The Iterative Next Neighbor re-Gridding (INNG) algorithm [1] is a recently proposed accurate and efficient reconstruction algorithm for non-uniformly sampled k-space data when compared with the previously proposed gridding algorithms. In the INNG algorithm, iterative procedures are performed using large rescaled matrices with two constraints imposed at each iteration. The INNG algorithm is quite simple and leads to accurate reconstruction. It's primary disadvantages are that it requires multiple 2D-FFTs on a large rescaled matrix, and the whole algorithm cannot be started until data acquisition is complete. In this study, we present a new method to efficiently implement the INNG algorithm. In this method, although a few iterations are still required after data acquisition, most of the computations can be performed before k-space data acquisition based on the known k-space data coordinates of the acquired data. The newly proposed algorithm is referred to as 'Precalculated INNG (PINNG)' algorithm. The image quality of the PINNG algorithm is comparable to that of the original INNG algorithm. This newly proposed method considerably improves image reconstruction efficiency and speed while maintaining overall accuracy. While PINNG is useful in itself, it also has a significant potential for faster implementation of INNG-related algorithms, e.g., POCSENSINNG [2] and Partial Fourier Spiral Reconstruction (PFSR) [3].

Methods

The flow chart of the basic INNG algorithm [1] is shown in Fig.1. Here, s is a scaling factor and N x N is the target grid matrix size. In the basic INNG algorithm, after k-space data are distributed to a large rescaled matrix sN x sN (Fig.1 (a)), iterative procedures (the loop (b) → (c) → (d) → (e) → (b) (surrounded by the dashed line in Fig.1)) are performed with two constraints imposed at each iteration. Iteration is repeated until the difference between the updated image matrix (b) and the image matrix (b) at the previous iteration becomes sufficiently small.

The procedure of the basic INNG algorithm can be mathematically formalized. The following three sN x sN matrices are first defined: A is identical to the matrix Fig.1 (a); B is the 2D-FT of a rect function with amplitude 1 within the central N x N matrix; and C is defined to be zeros at the elements where the original data exist in Fig.1 (a) and ones at all other elements. Suppose that D is the Fourier data matrix sN x sN after a sufficient number of iterations using the basic INNG algorithm. D can be expressed as: $D = ((\cdots\cdots((((A*B\times C+A)*B\times C+A)*B\times C+A)\cdots\cdots)*B\times C+A)$ [Eq.1], where $*$ and $\times$ represent a convolution and a pixelwise multiplication, respectively. We consider the following expression D' as an approximation of D: $D' = ((\cdots(A*B\times C+A)\cdots)*B\times C+A)+A*B\times C*\cdots*B\times C*B\times C\times E$ [Eq.2] where $E = ((\cdots\cdots((((C*B\times C+C)*B\times C+C)*B\times C+C)\cdots\cdots)*B\times C+C)$ [Eq.3]. E can be calculated before data acquisition if the k-space data coordinates are known. Only a small number of iterations are required to calculate D' after data acquisition, as seen in Eq.2. Note that the computational cost required for one iteration in D' after data acquisition is the same as that required for one iteration in the basic INNG algorithm. In the following, L_E denotes the number of iterations included in E. $L_{D'}$ represents the number of iterations performed after data acquisitions.

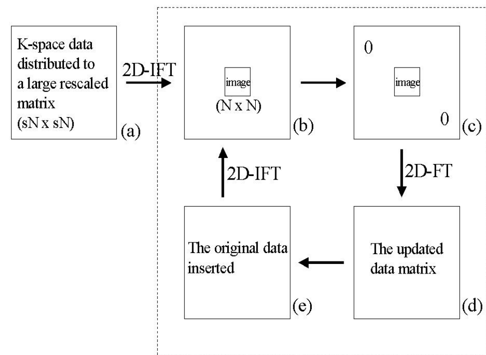

Fig. 1. A flow chart of the basic INNG algorithm

A computer simulation was first performed to test this algorithm. K-space data were calculated along 10 interleaved spiral trajectories from a numerical phantom (128x128 matrix) (Fig.2). Noisy data were also generated by adding Gaussian white noise to the ideal simulated complex data (the mean of the noise was 0 and the standard deviation of the noise was 20% of the average magnitude of the ideal data.). Both ideal and noisy data were reconstructed using the PINNG algorithm. The basic INNG algorithms were also used for comparison. In the PINNG algorithm, L_E was fixed at 100 and the following values of $L_{D'}$ were tested: 2, 3, 4, 5, 19, and 29. A scaling factor s was set to 8 in both the PINNG and basic INNG algorithms. The root mean square (RMS) error was measured for each reconstructed image. The image SNR was also measured for each image from noisy data.

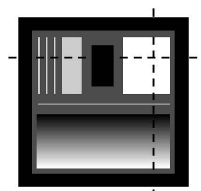

Fig. 2. A phantom

Results

Figure 3 shows the image profiles from the ideal data reconstructed using the PINNG algorithm with ($L_{D'}$, L_E) = (4,100). The corresponding lines are indicated in Fig.2. There are no observed profile distortions in Fig.3. The RMS error was 6.49%. Table 1 summarizes the RMS errors and the image SNR. For the ideal data, the RMS errors were decreased as $L_{D'}$ was increased in the PINNG algorithms with L_E fixed at 100. The RMS errors of the PINNG algorithms with ($L_{D'}$, L_E) = (19, 100) and (29, 100) are comparable to that of the basic INNG algorithm after 101 iterations. For noisy data, the RMS errors were not significantly changed with $L_{D'}$ greater than 3 in the PINNG algorithms when L_E was fixed at 100. There was no significant difference in image SNR among all the PINNG algorithms.

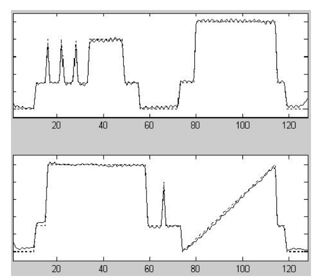

Fig. 3. Image profiles using the PINNG algorithm

Algorithm \ Data		Ideal data RMS error (%)	Noisy data RMS error (%)	Noisy data SNR
PINNG (L_E=100)	$L_{D'}$ = 2	31.58	33.95	22.5
	$L_{D'}$ = 3	13.57	19.66	22.8
	$L_{D'}$ = 4	6.49	16.00	22.7
	$L_{D'}$ = 5	6.14	15.94	22.6
	$L_{D'}$ = 19	4.96	15.79	21.9
	$L_{D'}$ = 29	4.45	15.81	21.5
Basic INNG	101 iterations	4.54	15.14	22.5
	30 iterations	10.84	13.53	42.6

Table 1. RMS error and SNR comparison

Discussion and Conclusions

As shown in Fig.3, the PINNG algorithm reconstructs images of high quality with only four iterations after data acquisition when s = 8 under the condition that sufficient iterations are pre-performed from a-priori k-space trajectory information. The number of iterations required for the basic INNG algorithm to achieve the same RMS error (6.49%) was 57. Thus, the computational cost of PINNG with ($L_{D'}$, L_E) = (4,100) relative to that of the basic INNG is 4/57 = 7.0%. As mentioned, we have computed D' in Eq.[2] instead of D in Eq.[1] although they are not equal to each other in a rigorous mathematical sense. However, our results suggest that D' is a good approximation of D.

The newly proposed 'PINNG' algorithm has been presented as a method of accurate and efficient reconstruction for non-uniformly sampled k-space data. It considerably reduces the number of iterations after data acquisition from the previously proposed basic or facilitated INNG algorithms. Although the PINNG algorithm still requires a small number of iterations following completion of data acquisition, most of them can be calculated a-priori. The PINNG algorithm is a new, fast reconstruction technique alternative to the INNG algorithms while maintaining their accuracy. Furthermore, it is considered that the computational costs of other INNG-related algorithms [2,3] may be significantly reduced using the methods similar to the PINNG algorithm described here.

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References [1] Moriguchi H, et al. Proc ISMRM 2003. p1066. [2] Moriguchi H, et al. ISMRM 2003. Late-breaking MR abstract. [3] Moriguchi H, et al. Proc ISMRM 2003. p1064.