

# Phase-Constrained Reconstruction of Sensitivity-Encoded MRI Data with POCSSENSE

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**Introduction:** Using phase constraints in sensitivity-encoded data reconstruction promises a number of benefits, including improved image quality, signal-to-noise ratio (SNR) increase and increased speedup factors [1, 2]. Matrix formulation of the phase-constrained reconstruction often requires explicit construction of large matrices, which is a computationally and memory demanding procedure. We outline a method for phase-constrained reconstruction of sensitivity-encoded data based on POCSSENSE approach [3, 4]. Our method avoids construction of large matrices, is applicable to arbitrary  $k$ -space trajectories, algorithmically simple and computationally inexpensive.

**Theory:** In method of projections onto convex sets (POCS), all available data and constraints are considered as convex sets in Hilbert space consisting of all possible images. POCS-based algorithms find solution by a sequence of projection operators, each mapping the current image estimate onto the nearest point of corresponding convex set. In POCSSENSE, the solution is obtained as a point with minimized weighted least squares distance from all the applicable convex sets. The new phase-constraining method is designed by introducing into POCSSENSE an additional projection operation onto a set of images with predefined image phase (see Algorithm). The resulting algorithm first projects in parallel the current image estimate onto available data (Step 1). The projection is accomplished by resetting  $k$ -space samples at the trajectory positions to the original values. Then, the results of projections are combined in optimal SNR way (Step 2), and the predefined phase  $\phi(r)$  is assigned to the image estimate (Step 3). Finally, the projection onto additional convex sets such as the one defined by object support area or maximum admissible intensity value is taken (Step 4).

## Algorithm. Phase-Constrained POCSSENSE Reconstruction

Notations:  $s_i$  – coil sensitivities,  $i=1, \dots, N_C$ , \* – complex conjugation

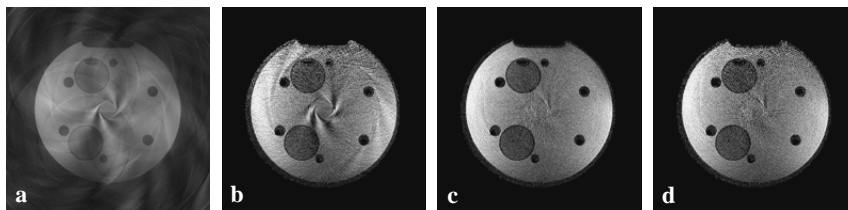
$$\text{Step 1: } g_i^{(n+1)} = P_i^{\text{data}}(s_i f^{(n)}), i=1, \dots, N_C \quad (\text{Project in parallel onto data convex sets})$$

$$\text{Step 2: } t^{(n+1)} = \left( \sum_{i=1}^{N_C} g_i^{(n+1)} s_i^* \right) / \left( \sum_{i=1}^{N_C} |s_i|^2 \right) \quad (\text{Combine the projection results})$$

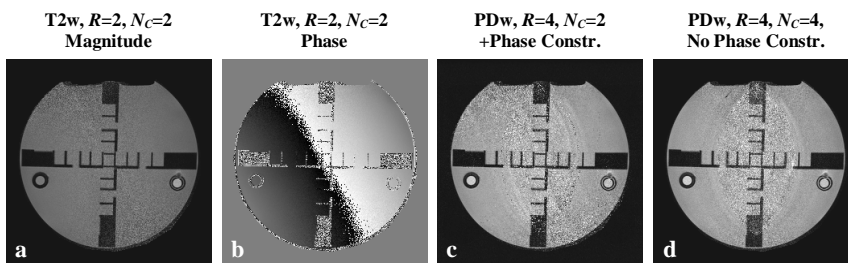
$$\text{Step 3: } z^{(n+1)} = |t^{(n+1)}| e^{i\phi(r)} \quad (\text{Constrain to the phase estimate})$$

$$\text{Step 4: } f^{(n+1)} = P_A(z^{(n+1)}) \quad (\text{Project onto additional convex sets})$$

**Methods:** All imaging experiments were performed on 1.5T GE SIGNA scanner (GE Medical Systems, Milwaukee, WI). Spiral dataset (18 interleaves, 2048 points per interleave, image matrix 192-by-192) was acquired using 4-element phase array ( $N_C=4$ ). The undersampled dataset was constructed by picking up every third interleave resulting in reduction factor  $R=3$ . The non-Cartesian POCSSENSE [4] was implemented using standard gridding with a Kernel-Bessel kernel ( $L=3, B=14.1372$ ) [5] along with Pipe's density function [6]. The sensitivity maps were estimated by polynomial smoothing the normalized images obtained from the fully sampled data. Additionally, the new method was applied to double echo fast spin echo (DE FSE) phantom data. The double-contrast data were used to obtain two separate datasets, the first is PD-weighted (PDw), the other T2-weighted (T2w). The idea was to extract the phase from one contrast image reconstructed from undercritically ( $R < N_C$ ) reduced data and, assuming phase consistency of DE FSE data, use it in reconstruction of the other contrast image from data sampled with overcritical reduction factor.



**Figure 1.** Phase-constrained reconstruction of spiral data ( $R=3$ ). **a:** image formed by sum-of-squares of two coil images; **b:** reconstruction using two coil data; **c:** reconstruction using four coil data; **d:** reconstruction using two coils and constraining to the phase of image (c). Arbitrary large number of iterations ( $N_i=100$ ) was applied in each case. Images (c) and (d) are almost identical except for increased noise level in (d). The residual artifacts in the image center are due to the coil configuration, non-optimal for reconstruction of undersampled spiral data at almost maximal reduction factors. Note that reconstruction is accomplished with no use of  $k$ -space conjugate symmetry property as a number of interleaves was even.



**Figure 2.** Phase-constrained reconstruction of sensitivity-encoded DE-FSE phantom data. The overall reduction factor is  $R=3$  while using only two channels ( $N_C=2$ ) for reconstruction. Reconstruction of image (c) from overcritically sampled PDw data ( $R=4$ ) is accomplished while constraining to the phase (b) extracted from T2w image (a) reconstructed from data with maximal reduction factor ( $R=2$ ). Reference image reconstructed with the same reduction factor ( $R=4$ ), but with higher number of coils ( $N_C=4$ ) is shown in (d).

**Results:** Figure 1 demonstrates results of phase-constrained POCSSENSE reconstruction of overcritically sampled spiral data. Figure 2 illustrates reconstruction of DE-FSE images from data with overall reduction factor in overcritical range ( $R=3, N_C=2$ ).

**Discussion:** We presented a method for phase-constrained, POCS-based reconstruction of sensitivity-encoded MRI data. Our results demonstrated that the knowledge of image phase estimate permits undersampling the  $k$ -space with overcritical reduction factors ( $R > N_C$ ) resulting in imaging speedup above the traditional limit  $R=N_C$ . The phase constraining (Step 3 in Algorithm) is just resetting the image phase to the predefined values, and, hence, it does not change method complexity. The practical aspects related to large matrix sizes in the matrix formulation may impede practical application of phase-constrained reconstruction of sensitivity-encoded data. Our method eliminates the need to form large matrices and avoids matrix inversion. The practical approaches to obtain phase estimates for improved reconstruction depend on a particular application. As we demonstrated, multiple contrasts FSE data could be used for phase extraction. In partial-Fourier method, the phase could be extracted from  $k$ -space center that should be sampled more densely than the rest of the  $k$ -space. Self-calibrating sensitivity encoding is a good choice for combination of partial Fourier and parallel imaging, as the  $k$ -space center is always critically sampled to assess coil sensitivities.

## Acknowledgments:

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## References:

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