Fitting Coil and Data in Parallel Imaging

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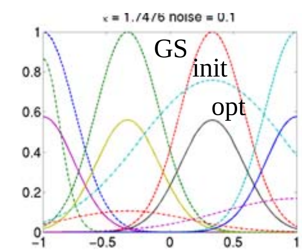
Introduction: Parallel imaging reconstructions using SMASH and SENSE require an estimate of the coil spatial sensitivities[1,2,3]. These vary depending upon the coil loading and so must be determined on a patient specific basis, by acquiring an extra reference scan. The maximum speed-up factor for parallel imaging is the number of coils, but speed-up factors less than this are frequently used. This leaves some extra information which may be sufficient to obtain the coil sensitivity profiles. Drawbacks of a reference scan include the extra scanning time and possible motion between the reference and speeded-up scan. Here, we investigate the use of directly fitting image data *and* parameters for the coils to the data acquired with speed up.

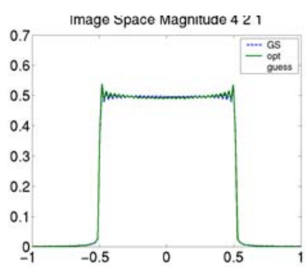
Method: We consider the equations both in image and k-space. The, unknown, image at x is denoted *r(x)*, its Fourier transform *R(k)*. The collection of all measured coil images is denoted *S_m(k)*, respectively *s_m(x)*. The coils are written *c(x)*, *C(k)*. Then, the standard SENSE/g-SMASH equations expressed as a *linear* Least-Squares become: minimise over **r** the cost |*s_m* – *c*.***r***| or minimise over **R** the cost |*S_m* – *C*****R***| where * denotes convolution, all in matrix-vector notations. We generalise these equations by making the coil depend on some parameters ***p***, so that the problem becomes: Minimise over ***r,p*** the cost |*s_m* – *c*(***p***)***r***| or minimise over ***R,p*** the cost |*S_m* – *C*(***p***)****R***|. In general, the coil as function of ***p***, *c*(***p***) or *C*(***p***), will be a nonlinear function, and the problem becomes a nonlinear Least-Squares. We solve the equations using the Matlab Optimisation Toolbox. In our examples, we modeled the coil sensitivity profiles as Gaussians: $c(x,\mathbf{p}) = p_1 e^{-(x-x_0)^2/(2p_2^2)}$, $\mathbf{p}=[p_1, p_2]$. This seems justified for the linear coil array we have simulated here.

The standard linear Least Squares equations involve a matrix built out of the coil sensitivities. The condition number can be computed from this matrix and we get $\kappa(\mathrm{FOV}, p) = \max_{x\in \mathrm{FOV}}\left(\sqrt{\sum_{coilsj}\left|c_j(x,p)\right|^2}\right) / \min_{x\in \mathrm{FOV}}\left(\sqrt{\sum_j \left|c_j(x,p)\right|^2}\right)$. Note that this is a function of the field of view and parameters ***p***, which can also be separately minimised for optimal coil positioning. This condition number also appears to have an effect on the convergence in our nonlinear optimisations.

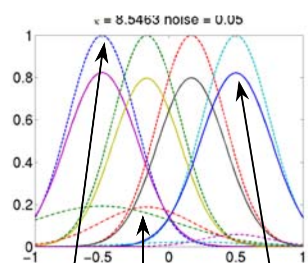
Results: We ran simulations using Gaussian coil profiles and a rect image as the Gold Standard. The k-space version of the cost function was used with different speed-up factors 'ns' simulated. Noise was added to the data. The figures show results with a speed-up factor of 2 and 4 coils at two different starting locations.



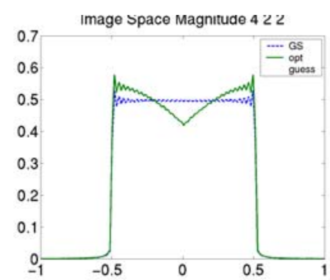
Coils, random starting estimate ns = 2 κ = 1.8.



Corresponding reconstructed magnitude image.



Coils, ns =2, κ = 8.5.
GS Initial Optimised



Corresponding magnitude.

The starting estimates for the coil widths and heights were randomly chosen positive numbers, and the starting estimate for the k-space was a constant. The coil locations are assumed to be known. In the figures, the dash-dotted line shows the initial estimates, the dashed line the gold standard, and the full line the result. For the coils spread over the whole FOV (left), the simulation finds the correct coil and image data. When the coils are clustered towards the centre of the FOV, the condition number is higher and the solution is less good (right).

Discussion: It is possible to reconstruct image data and coil parameters, where the results depend on: **---**the coil position (i.e. the condition), as can be seen in the figure, this is one of the main factors, **---**the speedup factor (ns = 3 was tried with similar results). In a more general case, the coils would be modeled for example by parametrising them by some components in k-space. Note that the equation allows a modulation factor, but it was found that fixing the coil positions constrains the problem so that we get convergence to the correct solution. Thus, using more parameters for the coils is possible in the sense that there are enough equations to have more unknowns, but care is needed in choosing what should be considered a free parameter. This study, even with so far simulation results, gives an insight about which parameters are important to consider, and when the problem is soluble, as clearly completely unknown parameters would be underdetermined.

References: [1] Pruessman et al *Magn Reson Med* **42**(5):952-62 (1991).[2] Sodickson et al. *Magn Reson Med* **38**(4):591-603. (1997)
[3] Bydder et al, *Magn Reson Med* **47**(1):160-70 (2002).

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