An Optimal SENSE Algorithm in Presense of Sensitivity Noise Using Total Least Squares

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Abstract

Traditional parallel imaging using SENSE is non-optimal in presence of sensitivity noise. An optimal algorithm for noisy sensitivity is presented, based on total least squares (TLS). The solution is the minimizer of a midly non-linear objective function, and a fast, efficient algorithm is presented to find it. Our approach comprises a novel TLS algorithm which is more appropriate than general TLS methods, while eschewing expensive computations required in the latter. Preliminary results suggest more than 15 dB improvement in SNR in some cases compared to standard SENSE.

Introduction

Parallel imaging using SENSE is based on a least squares inversion of the over-determined system corresponding to the sensitivity-encoding matrix E

$$y = Ex + n$$

with E assumed full column rank [1,2]. Here x is the desired image, y a concatenation of outputs from various coils, and n is white Gaussian noise assumed to be independent and identically distributed. While this inversion is optimal in the presence of noise in coil outputs, it fails to consider the effect of errors in sensitivity, which also undergoes similar noise processes during measurement. Under these conditions traditional methods become non-optimal [3]. We present a new optimal TLS [4, Ch.4] algorithm for SENSE in presence of i.i.d. AWGN in the measurement of sensitivity maps.

Proposed Method

Standard TLS methods involve expensive computation of SVD of large matrices. In this paper we eschew this approach and instead derive a TLS solution from a maximum likelihood viewpoint. We show that the optimal TLS solution is given by the minimizer of a quasi-quadratic objective function under reasonable constraints. This allows us to use efficient quadratic programming techniques like Conjugate Gradients (CG) [4, Ch. 7]. In this paper we only consider rectilinear k-space sampling, reducing the problem size to R, the frequency down-sampling factor. Breaking the data vector x and into R sub-vectors and defining η as per

$$x \equiv [x_1^T \; \Lambda \; x_R^T]^T, \quad \eta_x = \sum_{r=1}^{R} |x_r|^2$$

Let L be the number of coils, and β be the ratio of sensitivity error to measurement error. The TLS solution is then given by

$$\hat{x} = \underset{x}{\operatorname{argmin}} \left\{ (y - Ex)^H (I - Q(x))(y - Ex) \right\}$$

Where $Q(x)$ is a data-dependent matrix given via a diagonal matrix $B(x)$

$$Q(x) = \begin{bmatrix} B(x) & \Lambda & B(x) \\ \mathrm{M} & \mathrm{O} & \\ B(x) & & B(x) \end{bmatrix}, \quad (B(x))_{i,i} = \frac{\beta^2 (\eta_x)_i}{1 + \beta^2 L (\eta_x)_i}$$

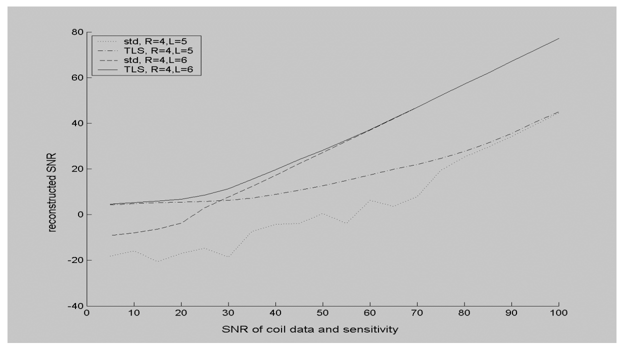

Figure 1: SNR performance of TLS and standard SENSE

The non-linearity above enters the equation only via the “aliased energy” term η. Since successive estimates of the minimizer are not expected to yield widely varying values (since β is usually small) for this mildly non-linear energy term the objective function can be efficiently solved by a fast modified Newton-type algorithm. The gradient of the objective is readily available in closed form, and can be used to speed up convergence. As a result, the complexity of the TLS algorithm is only slightly greater than O(N).

Preliminary Results

Sensitivity of circular coils were simulated from the Biot-Savart Law. Noise in both sensitivity and coil outputs was assumed equal. The performance of our TLS algorithm is evaluated for $[R,L] = 4,5$ *and* $3,4$ in fig 1, which shows reconstructed versus original SNR values. It is noteworthy hat standard SENSE is always trailing TLS SENSE in terms of SNR, in every case. Around 15 dB improvement over standard SENSE (i.e.1/100th of noise energy) was observed at high noise, converging at low noise as expected. This impressive gain is further demonstrated in Fig 2, which shows a high-noise scenario wherein the standard SENSE is almost unusable, but TLS-SENSE proves quite effective.

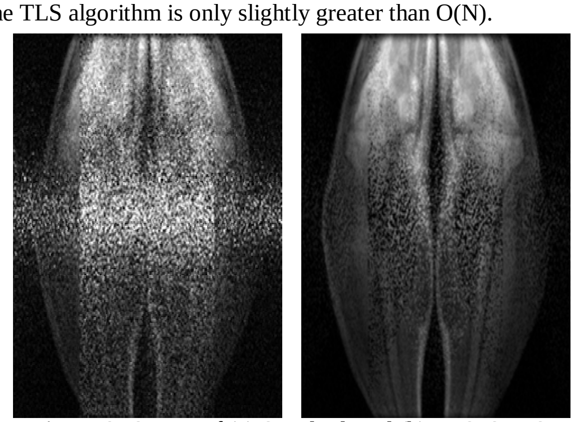

Figure 2: Output of (a) Standard, and (b) TLS_SENSE

Conclusion

This paper addresses the problem of obtaining an optimal solution to the parallel imaging reconstruction problem in the presence of both measurement and sensitivity noise. The solution is shown to be the minimizer of a quasi-quadratic objective function which may be solved efficiently via a modified Newton-type algorithm with gradient information.

References

1. Preussmann KP, *et.al. MRM* 42:952-962, 1999.
2. Preussmann KP, *et.al. MRM* 46:638-651, 2001.
3. Golub G, Van Loan C, *Matrix Computations*, 1996.
4. Ake Bjorck, *Numerical Methods for Least Squares Problems*, SIAM press, 1996.