Backus-Gilbert Regularization for SENSE Imaging

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Abstract

A new regularization technique for SENSE image reconstruction is considered. This technique, Backus-Gilbert regularization, constrains both the expected deviation of the reconstructed image from the true image and the stability of the image. This regularization requires no prior knowledge of the likelihood of the image which is crucial for the application of linear regularization. Our reconstruction applied to simulated data acquired with a four-element phased array coils and reduction factor (R=4), and sensitivity maps calculated by Biot-Savart, show that there is a reasonable trade-off between signal and aliasing for a given weighting factor.

Introduction

SENSE (SENSitivity Encoding) is a semi-parallel imaging technique which uses multiple RF coils to reduce imaging time by reducing the FOV. Unfolding the aliased images requires inverting a matrix equation which can be sensitive to perturbation if the sensitivity is nearly singular. The latter circumstance is often the case for high reduction factor.

For a given pixel ρ and reduction factor R, the signal by the j^{th} coil is given by,

$$I_j(\rho) = \sum_{k=0}^{R-1} S_j(\rho + kD) M(\rho + kD) + n_j \qquad [1]$$

where j runs over the coil number, $S_j(\rho)$ is the sensitivity of coil j at pixel ρ, M(ρ) is the unknown spin density, n_j is the noise in each channel and D=FOV/R. We can invert equation 1 to solve for M. In matrix form this solution can be written as,

$$M = S^{-1}(I - N) \qquad [2]$$

The solution becomes unstable and unrealistic if the sensitivity is close to singular. Thus it is necessary to incorporate further information about the likelihood of the solution in order to single out a useful and stable solution. Regularization techniques often introduce another quantity (a functional of the spin density) which describes smoothness or stability of the desired function and seeks a solution which minimizes this quantity along with the functional whose minimum corresponds to satisfying equation 2 [1,2].

Suppose $\hat{M}$ is the reconstructed spin density and M is the unknown true spin density, the central idea of any regularization is to minimize the quantity [2],

$$A(\hat{M}) + \lambda B(\hat{M}) \qquad [3]$$

where $A(\hat{M}) > 0$ and $B(\hat{M}) > 0$ are two positive functionals. For SENSE, $A(\hat{M}) = |S\hat{M} - I|^2$, and $B(\hat{M})$ is defined depending on the quantity we would like to minimize. In Backus-Gilbert regularization B is chosen as a measure of how much the solution varies within the measurements along with constraint that the expected deviation of the spin density from the true M to be minimum. We can write the relationship between $\hat{M}$ and M as [2]

$$\hat{M}(\rho) = \sum_{\rho'} \delta_{\rho\rho'} M(\rho') \qquad [4]$$

where δ is a resolution function, for a perfect reconstruction it is a Dirac Delta function. In this regularization technique $B(\hat{M})$ is

$$B(\hat{M}) = Var[\hat{M}(\rho)] + \beta \sum_{\rho} D^2 (\rho - \rho')^2 \hat{\delta}^2_{\rho,\rho'} \qquad [5]$$

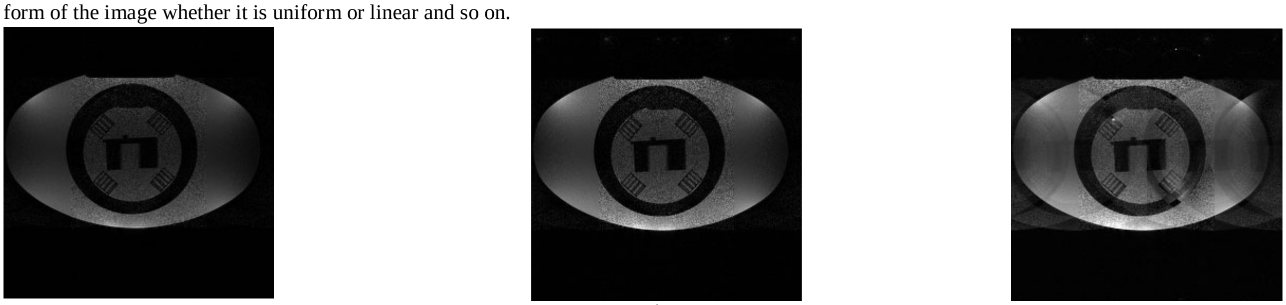
Here the first term measures how much the solution vary as the data varies within different measurements and the second term guages the expected deviation of the measured spin density from the true value and β is a weighting parameter. Minimizing $A(\hat{M}) + \lambda B(\hat{M})$, we will have

$$\hat{M} = ((S^{+}S + \lambda E)^{-1} \cdot S^{+} + \lambda q) \cdot I \quad \text{where} \quad q(\rho) = \frac{[W(\rho) + \beta S']^{-1} \cdot R}{R \cdot [W(\rho) + \beta S']^{-1} \cdot R} \qquad [6]$$

$W_{ij}(\rho) = \sum_{\rho} D^2 (\rho - \rho')^2 \delta^2_{\rho\rho'} S_i(\rho') S_j(\rho')$ is a function of the second moment of the resolution function, $R_i = \sum_{\rho} S_i(\rho)$ is a function of the sensitivity of the coils which is determined by the minimization principle , S' and E are the covariance and identity matrices, respectively.

Result and Discussion

Our result based on phantom image obtained from a Philips Infinion machine and a calculated sensitivity for four surface coils and maximum reduction factor R=4, shows that there is a reasonable trade-off between signal and aliasing for $\lambda = 10^{-2}$ and β=1. There, is in fact, an 8.6% and 7.9 % difference in the maximum and mean signal between figure 1b and figure 1a. Unlike linear regularization (first order and higher), this regularization technique does not require a priori judgment of the form of the image whether it is uniform or linear and so on.



a) b) c)

Fig. 1. Image reconstructions using equation 6 for four surface coils with positions two at the top and two at the bottom for R=4. a) with out regularization, b) with Backus –Gilbert regularization, with $\lambda = 10^{-2}$, and c) for λ=1.

Reference

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