Improved Preconditioning for the Non-Cartesian SENSE Reconstruction with Regularization

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Abstract

Regularization has proven to be beneficial to the reconstruction of sensitivity-encoded acquisitions. Applied to non-Cartesian SENSE imaging, it permits to stabilize the iterative procedure for solving the underlying inverse problem, but at the same time it affects the convergence behavior and thus the running time of the reconstruction. In this work, the preconditioning usually employed for acceleration is shown to become less effective in conjunction with a regularization. A modification to it, which involves an adaptation to the selected regularization, is proposed and demonstrated to preserve the speed of convergence achieved without regularization to date.

Introduction

The reconstruction of sensitivity-encoded non-Cartesian acquisitions may be reduced to solving a large linear system of equations (LSE) with iterative methods [1]. Preconditioning is a common technique to accelerate the convergence of these methods. A sampling density compensation and a coil sensitivity normalization have been proposed for this purpose in the present context [1]. Regularization is a well-known technique to trade off accuracy against stability in inverse problems in general. It has mainly been employed in the direct reconstruction of sensitivity-encoded Cartesian acquisitions so far to limit noise amplification at the expense of aliasing suppression [2,3]. While it is in principle applicable to the non-Cartesian SENSE reconstruction as well, it turns out to slow down the convergence.

Theory

The LSE to be solved iteratively reads [1]

$$E^H D E \underline{x} = E^H D \underline{m} , \qquad \text{with } E_{(\gamma,\kappa)\rho} = s_{\gamma\rho}\, e^{ik_\kappa r_\rho} .$$

D denotes an optional diagonal matrix for sampling density compensation, $\underline{x}$ the unknown image, $\underline{m}$ the measured raw data, and s the coil sensitivity. Incorporating regularization into this equation yields

$$(E^H D E + \alpha R^{-1})\underline{x} = E^H D \underline{m} ,$$

where α represents a scalar weighting factor and R a diagonal matrix. This work makes no assumption about the choice of α and R, of which various have been put forward over recent years.

The application of preconditioning with a diagonal matrix I leads to

$$(IE^H DEI + \alpha IR^{-1} I)(I^{-1}\underline{x}) = IE^H D\underline{m} .$$

Instead of using a coil sensitivity normalization only, it is proposed to adapt the preconditioning to the regularization by setting

$$I_{\rho\rho} = ((\sum_\gamma | s_{\gamma\rho} |^2) + \alpha R_{\rho\rho}^{-1})^{-0.5} .$$

The regularization term may then be rewritten as

$$(\alpha IR^{-1}I)_{\rho\rho} = (1 + \alpha^{-1} R_{\rho\rho} \sum_\gamma | s_{\gamma\rho} |^2)^{-1} .$$

In this way, the evaluation of the matrix-vector product, the dominant operation of each iteration, reduces to the calculations described in Ref. 1, but with a modified intensity correction, and the subsequent addition of the appropriately weighted vector.

Methods

The effectiveness of the described preconditioning was tested on a twofold subsampled spiral acquisition of a transverse cross section of a brain with a six-element head coil. Without regularization, changes in the intermediate images were no longer noticeable after two iterations.

The coil sensitivities were calculated from a separate low-resolution reference scan. It also provided information on the expected signal power, which served as a priori knowledge for the regularization. The weighting factor α was set to 1.0.

Results

The images in Fig. 1 illustrate that more than six iterations are required to suppress aliasing with the conventional preconditioning. By contrast, two iterations with the proposed preconditioning are sufficient to obtain a comparable image quality. Hence, an acceleration by at least a factor of three is achieved in this case.

Conclusion

By adapting the preconditioning to the selected regularization, it is possible to counteract the effect the regularization has on the speed of convergence. A regularization may thus be incorporated into the non-Cartesian SENSE reconstruction without prolonging the running time.

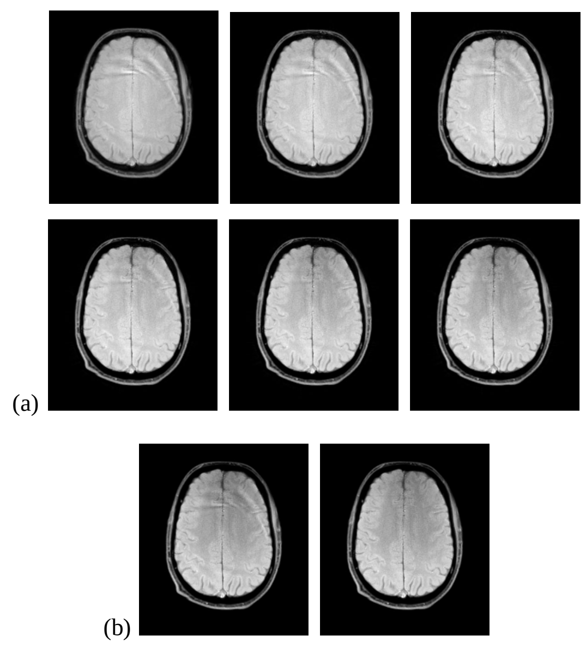

Fig. 1. Progression of the non-Cartesian SENSE reconstruction with regularization using (a) the conventional and (b) the improved preconditioning.

References

1. Pruessmann KP, et al., Magn Reson Med 2001; 46:638-651.
2. King KF, et al., Proc ISMRM 2001; 1771.
3. Tsao J, et al., Proc ISMRM 2002; 739.