Relative SENSE (rSENSE) and Sensitivity Map Calculation from k-Space Reconstruction Coefficients (SMACKER)

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Introduction: The determination of accurate coil sensitivity profiles is crucial for image reconstruction using SENSE. In classical SENSE the profiles are determined in a so called calibration scan by division of coil array element images by a body coil image sampled with full FOV [1]. However, as this relies on the availability of a body coil, there are many situations imaginable where this is not possible. In such cases an autocalibrating SENSE technique such as mSENSE [2] can be used. In this abstract a different approach, rSENSE (relative SENSE), is presented: Instead of a body coil, one of the array elements itself is used as a reference and sensitivity maps relative to this element are calculated. The image reconstructed by SENSE in this case then corresponds to a fully sampled image taken with the corresponding array element. Thus it is possible to reconstruct single-element images while preserving the phase information contrary to mSENSE. To obtain a combined image the procedure is repeated for each element in the coil array and the separate images are combined by, for example, a sum-of-squares.

We further present a SMASH-based [3,4] method to calculate full FOV relative sensitivity maps from few lines in inner k-space that do not show the problem that the sensitivity map ends at object boundaries which is common to all methods determining sensitivity information by division of low resolution images. This method shows, in an experimental manner, the correlation between SMASH-coefficients and sensitivity maps already theoretically described e.g. in [5].

Theory: The relevant unfolding step in cartesian SENSE is described by the equation

$$\boldsymbol{v} = U\boldsymbol{a} \quad \text{with} \quad U = (S^H \Psi^{-1} S)^{-1} S^H \Psi^{-1}$$

$\boldsymbol{a}$ represents the folded and $\boldsymbol{v}$ the unfolded image data while the unfolding matrix U is composed of the sensitivity data S (obtained by division of coil array element images by a body coil image) and the noise information Ψ. If we replace S in this equation by sensitivity values obtained by division of the images of all array elements by one particular element we obtain, as a result of SENSE reconstruction, an image which corresponds to a fully sampled image from the particular array element with correct phase information. A combined image can be obtained by repeating the procedure and successively using each array element as a divisor in sensitivity calibration. Another possibility to obtain such relative sensitivity information which we called SMACKER (sensitivity map calculation from k-space reconstruction coefficients) is described in the following. We first consider the basic equation for k-space data $\boldsymbol{m}$ of array element γ in two-dimensional Fourier-Imaging (c is the spin density of the object and s_γ are the coil sensitivities; relaxation effects are neglected):

$$m_\gamma(k_x, k_y) = \iint_{\text{FOV}} c(x,y)\, s_\gamma(x,y)\, e^{-i(x k_x + y k_y)}\, dxdy$$

In SMASH, for example, missing k-space information $\boldsymbol{m}$ is calculated from a weighted combination of neighbouring k-space data. In a somewhat more general manner we can write:

$$m_\gamma(k_x, k_y) = \sum_{\lambda \neq \gamma, m, n} w_\gamma(\lambda, m, n)\; m_\lambda(k_x + m\Delta k_x, k_y + n\Delta k_y) + \sum_{(m,n)\neq(0,0)} w_\gamma(\gamma, m, n)\; m_\lambda(k_x + m\Delta k_x, k_y + n\Delta k_y)$$

A comparison of the two equations under the assumption that they hold for an arbitrary spin density distribution c leads to:

$$s_\gamma(x,y) = \sum_{\lambda \neq \gamma, m, n} w_\gamma(\lambda, m, n)\; s_\lambda(x,y)\, e^{-i(x m \Delta k_x + y n \Delta k_y)} + \sum_{(m,n)\neq(0,0)} w_\gamma(\gamma, m, n)\; s_\lambda(x,y)\, e^{-i(x m \Delta k_x + y n \Delta k_y)}$$

or:

$$\left(1 - \sum_{(m,n)\neq(0,0)} w_\gamma(\gamma, m, n)\; e^{-i(x m \Delta k_x + y n \Delta k_y)}\right) s_\gamma(x,y) - \sum_{\lambda \neq \gamma}\left(\sum_{m,n} w_\gamma(\lambda, m, n)\;\; e^{-i(x m \Delta k_x + y n \Delta k_y)}\right) s_\lambda(x,y) = 0$$

This is a system of linear equations for the sensitivities s_γ of the array elements at each point in image space. If the system had full rank the only possible solution would be the zero solution for all sensitivity profiles. Thus, in reality, the equations have to be dependent and so it is possible to calculate all sensitivity profiles relative to the profile of one particular element from generalized k-space reconstruction coefficients.

Methods: A 1.5 T SIEMENS Sonata 4-channel system was used together with two elements of a CP Spine coil array integrated into the patient table and a two-element CP Body flex coil array (SIEMENS Medical Solutions, Erlangen, Germany). Standard Spin-Echo images of a quality phantom were acquired with TR 500 ms, TE 20 ms, FOV 25 cm, slice thickness 5 mm and a 256² matrix. The images were numerically reduced to a reduction factor of 2 and then reconstructed using the SMACKER-technique for the generation of sensitivity maps and the rSENSE algorithm. The k-space reconstruction coefficients used for SMACKER were calculated from the data of 24 densely sampled reference lines in central k-space.

Results: Figure 1 shows sensitivity maps for three array elements relative to the fourth one. The circle in the maps indicates roughly the circumference of the phantom. Figure 2 shows the reconstructed magnitude image of one array element using rSENSE with the sensitivity maps from figure 1 and a reduction factor of 2. Finally, figure 3 shows the sum-of-squares combination of the four single-element images each reconstructed with rSENSE.

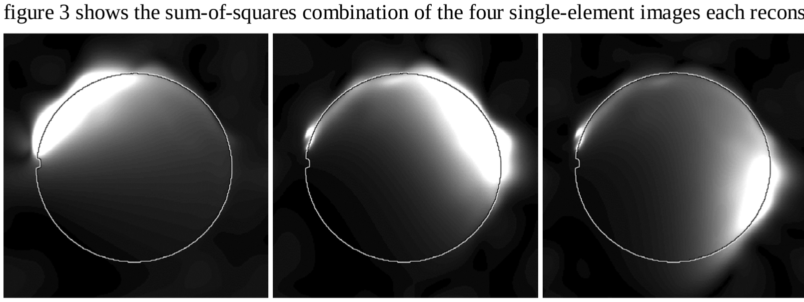

Figure 1

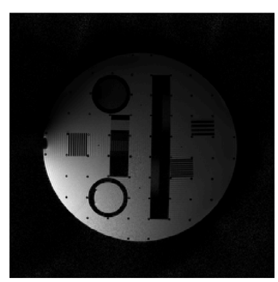

Figure 2

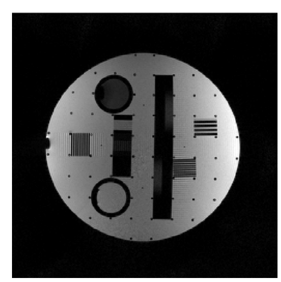

Figure 3

Discussion and conclusions: Within the phantom region the sensitivity maps present a smooth behaviour which extends somewhat beyond the object boundaries. In the outer regions of the FOV the quality of the sensitivity maps degrades. The image reconstructed using rSENSE and SMACKER with a reduction factor of 2 shows very good quality. There are practically no artefacts visible. It has thus been shown that it is possible in practice to use k-space reconstruction coefficients to generate sensitivity maps as already shown theoretically in [5]. Furthermore this technique comprises the possibility, contrarily to a standard SMASH or GRAPPA reconstruction, to calculate g-factors [1] for single-element reconstruction from the relative sensitivity maps.

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References: [1] K. P. Pruessmann et al., Mag. Reson. Med. 42:952-962 (1999); [2] D. K. Sodickson et al., Magn. Reson. Med. 38:591-603 (1997); [3] M. Griswold et al., Mag. Reson. Med. 47:1202-1210 (2002); [4] J. Wang et al., abstract 92, Wuerzburg Workshop on Parallel MRI, (2001); [5] J. Wang et al., abstract 769, Proc. Intl. Soc. Mag. Reson. Med. 9 (2001).