A new approach to Current Density Impedance Imaging (CDII)

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Abstract: We have developed a new approach (CDII) to Impedance Imaging using MRI. We use the MR imager to obtain two Current Density images [1]. We then use a simple mathematical expression to compute the gradient of the logarithm of conductivity, $\nabla\ln(\sigma)$, at each point in space in a region where two different current density vectors have been measured. From the images of the gradient of the logarithm of conductivity, $\nabla\ln(\sigma)$, we will be able to obtain images of $\ln(\sigma)$ by integration and of σ by *a priori* knowledge of the conductivity at a single point in the object. The CDII method was tested on data from an analytical model of a simple conductivity distribution with added Gaussian noise. It was also tested in a phantom constructed from a conductive tissue mimicking gel containing a saline filled cavity.

Introduction: Imaging bio-electric conductivity has long been the goal of Electrical Impedance Tomography (EIT). Previous EIT methods generally used only boundary voltage and current measurements to create a conductivity image. Due to the ill-posedness of the EIT problem there are limitations to EIT resolution as we move away from the boundary. Lately, a new magnetic resonance Electrical Impedance Tomography (MREIT) method, based in CDI, has been proposed to overcome the ill-posedness of EIT [2]. The proposed method solves a non linear boundary value problem. The iterative solution to this problem is computationally intensive and requires measurement of CD at all points inside and on the boundary. In practice CDI measurements cannot be obtained in tissues producing low MRI signal (in the lung for example) or near severe susceptibility artifacts. Our new approach to CDII gives a simple explicit formula for the gradient of the logarithm of conductivity, $\nabla\ln(\sigma)$, based on measurements of two independent current densities, $\mathbf{J}_1$ and $\mathbf{J}_2$ in a region of interest in an isotropic conductive material. Thus no iterative solution and no knowledge of the CD or its concomitant magnetic field is required except in the region of interest. Additionally, there is no doubt as to the uniqueness of our value for $\nabla\ln(\sigma)$ provided the material is isotropic.

Methods: The mathematical model used for testing our method was a spherically symmetric isotropic conductivity

$$\sigma(r) = \sigma_0 \exp(-\beta/r) \text{ for } r>r0, \text{ otherwise zero.} \qquad (1)$$

That is, a spherical insulator in an infinite inhomogeneous conductor. The current density in this model approached a constant, $\mathbf{J}_0$, at infinity and was tangent to the sphere at $r = r_0$. Poisson's equation, $\nabla\bullet\sigma\nabla u = 0$, was then solved analytically for the electric potential, u. The current density was found analytically from u using $\mathbf{J} = -\sigma\nabla u$. Samples of $\mathbf{J}$ over a rectangular grid were computed at 128^3 points. Independent Gaussian noise of variance 0, 2%, 5% and 10 % of $\mathbf{J}_0$ was added to these samples. This process was repeated with a value of $\mathbf{J}_0$ orthogonal to the first. The result was two, sampled, possibly noisy, current densities, at 128^3 points in the vicinity of an insulating sphere. Clearly, in the noiseless case, $\mathbf{J}_1$ and $\mathbf{J}_2$ are zero inside the insulator and orthogonal to each other outside. The interior points were subsequently ignored. Next, values of the curl, $\nabla\times\mathbf{J}_1$ and $\nabla\times\mathbf{J}_2$, of $\mathbf{J}_1$ and $\mathbf{J}_2$ were computed numerically using a 3x3x3 Sobel template similar to that described in [1]. Values of the cross product $\mathbf{K} = \mathbf{J}_1\times\mathbf{J}_2$ were also computed. Finally the value of $\nabla\ln(\sigma)$ was computed using:

$$\nabla\ln(\sigma) = (1/|\mathbf{K}|^2)\,\{(\nabla\times\mathbf{J}_2\bullet\mathbf{K})\mathbf{J}_1 - (\nabla\times\mathbf{J}_1\bullet\mathbf{K})\mathbf{J}_2 + (\nabla\times\mathbf{J}_1\bullet\mathbf{J}_2)\,\mathbf{K}\} \qquad (2)$$

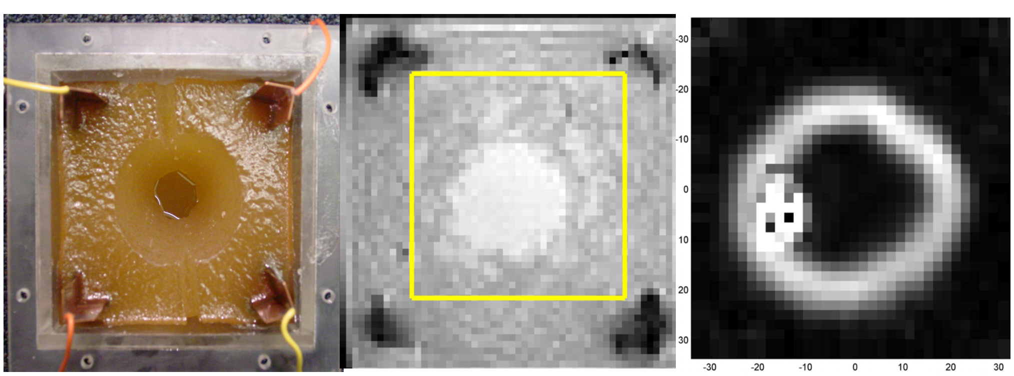
Note that $\mathbf{J}_1$, $\mathbf{J}_2$ and $\mathbf{K} = \mathbf{J}_1\times\mathbf{J}_2$ form a basis if $\mathbf{J}_1$ and $\mathbf{J}_2$ are not parallel. The value of $\nabla\ln(\sigma)$ was not computed at points where $|\mathbf{K}|$ fell below an arbitrarily chosen threshold. The formula (2) can be derived from Maxwell's Equations

$$\nabla\times\mathbf{E} = \mathbf{0} \text{ and the constitutive equation, } \mathbf{J} = \sigma\mathbf{E} \qquad (3)$$

The phantom used for experimental evaluation (left side of figure below) was rectangular (100x100x80mm) with two pairs of L shaped electrodes (10x10x80mm), one at each corner. A conical plug of gel of height 50mm and base 50mm was removed from the center. A ~spherical hole (radius ~20mm) was dug out at its apex and filled with saline solution of the same conductivity as that used when making the gel. Thus the conductivity in the hole was about 2 times higher than that in the gel. A standard LFCDI sequence [1] (1.9 mm cubic voxel, TR 3600, TE 50, Tc 34 ms, 2 averages) was used to measure the $\mathbf{J}_1$ and $\mathbf{J}_2$ created using diagonally opposing electrodes. Equation 2 was used to compute $\nabla\ln(\sigma)$ in a 70mm a side cube roughly centered on the spherical hole.

Results : The $\nabla\ln(\sigma)$ computed (using the right side of equation 2) from the $\mathbf{J}_1$&$\mathbf{J}_2$ is essentially identical to the left side of equation 2 even with the addition of 10% noise. In the figure below on the right is the magnitude $|\nabla\ln(\sigma)|$ in the region outlined in the middle MRI magnitude image. Only the CDI data from a central 70mm cube (outline middle) was used to calculate $|\nabla\ln(\sigma)|$. The direction of $\nabla\ln(\sigma)$ was generally inward. The cause of the artifact at 9 o'clock is unknown. The values of $|\nabla\ln(\sigma)|$ in the artifact were 4,600 mm^{-1}, 25x greater than the 180 mm^{-1} which is shown as white in the image.

Conclusions: The analytic model results demonstrated that CDII yields a stable solution. The experimental results demonstrated that CDII works in practice. Ideally the $|\nabla\ln(\sigma)|$ would be a circle, however the hole was roughly cut so some of the irregularity is real. The derivative templates used to evaluate Equation 2 have limited the resolution to ~ 6mm.



Left: Gel Phantom with conical plug removed. The dug out sphere is larger than the dark opening seen in the photo.. **Middle:** MRI magnitude image. **Right:** $|\nabla\ln(\sigma)|$ [in mm^{-1}] for the region outlined in the middle image.

References: **1**. G.C. Scott, M.L.G. Joy, R.L. Armstrong, and R.M. Henkelman, *Measurement of Non-Uniform Current Density by Magnetic Resonance*, IEEE Transactions on Medical Imaging, 10, 362-374, September, 1991. **2**.O.Kwon, E.J.Woo, J.R.Yoon, and J.K.Seo, *Magnetic resonance electrical impedance tomography (MREIT): simulation study of J-substitution algorithm*, IEEE Trans. Biomed.Eng., vol.48,no.2,pp. 106-167, 2002