Effects of Subpixel Integration on PERL Signal Simulations

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Synopsis

Numerical simulations of MRI signals generated by the PERL imaging sequence are presented. The PERL field encodes two dimensions simultaneously by employing a product of a **PER**iodic and a **L**inear spatial encoding field. The simulations provide a useful tool to test imaging sequences and the reconstruction procedures. One aspect of the simulations reveals severe aliasing of echoes unless the appropriate integration (analytic or numerical) is performed over subpixels.

Introduction

To test imaging sequences, identify artifacts, and to test reconstruction algorithms, it is useful to simulate the time dependent MRI signals that would arise from ideal phantom geometries. Likewise, numerical simulations of the PERL MRI signal have been extremely useful in testing our reconstruction methods. Care must be taken in the numerical approximations used in the calculation of the signal. In prior work [1-8], we proposed the use of the PERL field given by $B_p(x,y) = G_p y \sin(qx+\theta)$ where $\lambda = 2\pi / q$ is the spatial wavelength. If this field is used for a pre-encoding pulse of duration *T*, then the NMR signal acquired in the presence of a standard readout gradient, G_x, is given by: $S(t) = \iint \rho(x,y) \exp(i\gamma(G_p T y \sin(qx+\phi) - G_x t x))\, dx dy$ **(1)**. Numerical integration of Eq.(1) for a known phantom, $\rho(x,y)$, represents one method for simulating the PERL signal. An alternative method relies upon simulating the individual echoes that make up the PERL signal. *S(t)* may be decomposed [1-3] as given by: $S(t) = \sum_m s_m(t-t_m) \exp(im\phi)$ **(2)** where

$s_m(t-t_m) = \int \rho_m(x) \exp(-i\gamma G_x x(t-t_m))\, dx$ **(3)** with $t_m = mq / \gamma G_x$ and $\rho_m(x) = \int \rho(x,y) J_m(k_y y)\, dy$ **(4)** with $k_y = \gamma G_p T$.

The individual echoes, $s_m(t-t_m)$, may then be calculated from Eqs.(3-4). A key aspect for the numerical simulations revolves upon the discretization of $\rho(x,y)$. In earlier work [3], $\rho(x,y)$ was discretized into an array of points, converting the integrals in Eqs(1,3,4) into simple summations.

Discussion

From Fourier transform principles, it is well known that aliasing arises when the Nyquist criteria is not satisfied. Examination of Eq.(1) leads to the conclusion that only sampling of *x* will cause aliasing in time, *t*. If Δx represents the sample resolution along *x*, then aliased copies of the signal will occur at periodic intervals of $\Delta t = 2\pi / \gamma G_x \Delta x$. In normal MRI Fourier imaging, Δt corresponds to the acquisition time and this is usually not a problem. In PERL imaging, many echoes are collected at once with echo spacings given by $2\pi / \lambda\gamma G_x$. If a total of N echoes are collected, then in order to avoid aliased signals it is necessary for $\Delta x \le \lambda / N$. This can be a much finer resolution then expected. For example, if N~300 and λ=5cm, then Δx<0.17mm whereas Δx =1.17mm for 256 points across a 30cm FOV. If λ is an integer multiple of Δx then the aliased echoes will superimpose exactly with other echoes, providing the unexpected appearance that some of the echoes are identical. Under these conditions, we have observed that the reconstruction can return the correct image, which may give the impression that the starting signal was correct.

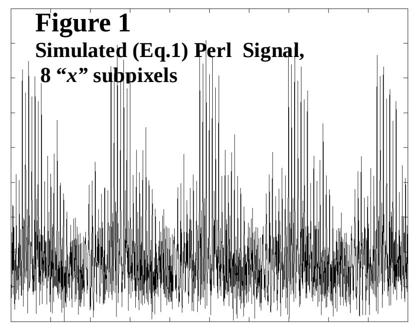

Figure 1
Simulated (Eq.1) Perl Signal, 8 "x" subpixels

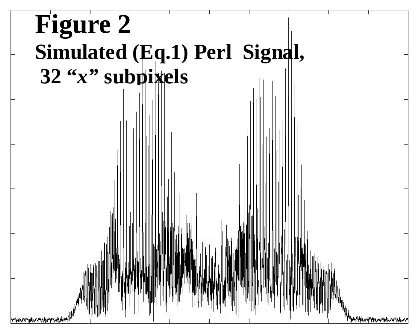

Figure 2
Simulated (Eq.1) Perl Signal, 32 "x" subpixels

Since simply increasing the grid size by a factor of 10 can increase the computation time 100 fold, we have employed the division of each pixel into subpixels. Over all the subpixels of a pixel, $\rho(x,y)$ is kept constant. The signal may be calculated from Eq.(1) or from Eqs.(2-4) for a finite number of echoes. Eq.(1) has the advantage in that the integration over subpixel space can be done analytically for *y*, while the integration over the *x* subpixels is done numerically by summation. Unexpectedly, it is possible to integrate Eqs.(3,4) analytically over both the *x* and *y* subpixels. Routines for numerical simulation of the signal have been developed in both Matlab and Mathematica software packages. Figures 1 and 2 show the results of a PERL signal simulation using Eq.(1). In Fig.1, the effect of echo aliasing is apparent even though 8 subpixels were used. Fig.2 shows that echo aliasing is eliminated with 32 subpixels.

References

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