Non-linear Magnetic Field Application for MR Imaging: Traveling Pulse Encoding

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Introduction. Traditionally, the acquisition of MR images is done using linear magnetic gradients to sample k-space, hence extracting information from the imaged object in the frequency domain. Even though parallel imaging techniques (via coil sensitivities) and non-Fourier excitation encodings make use of spatial information, this domain is seldom used for encoding during readout. The shift from the frequency to the spatial domain allows changing the acquisition paradigm with the purpose of obtaining images sampled at higher rates instead of at higher gradients. In this work we propose a simple way to exploit non-linear magnetic fields to encode the image, directly in the space domain.

Theory and Methods. If a magnetic field "moves" across the field of view (FOV), in a fixed direction, it dephases the spins as it travels. The resultant signal corresponds to the convolution of the image with a dephasing function. For simplicity, we show here the signal equation for one dimension.

Let $B_z(Vt\text{-}x)$ be a magnetic field in the z direction, moving along x with velocity V of finite length, i.e., $B_z(t=0,x) = 0, \forall x \notin [0, x_{max}]$. Let $\phi(x,t)$ be the phase induced on the spins, according to the integral of $\gamma B_z(Vt\text{-}x)$ in time. Since the pulse is moving, $\phi(x,t)$ will also be a function of Vt-x and the acquired signal s(t) (Eq. 1) is equivalent to the convolution shown in Eq. 2. The object m(x) can be recovered by deconvolving s(t) with h(t).

$$s(t) = \int_{-\infty}^{+\infty} m(x) \cdot e^{-i\phi(x,t)} dx = \int_{-\infty}^{+\infty} m(x) \cdot e^{-i\phi(V \cdot t - x)} dx \quad (1)$$

$$s(t) = m(Vt) * h(Vt), \quad h(Vt) = e^{-i\phi(Vt)} \quad (2)$$

As an example, let us consider a traveling Gaussian pulse of amplitude A, width σ and velocity V. The phase introduced by this pulse will be $\phi(x,t)$ given in Eq. 3 and the signal, s(t), in Eq. 4, which can be expressed as the convolution in Eq. 5. Fig 1 shows a simulation of this traveling Gaussian pulse. The upper graph shows the Gaussian pulse as it moves over the FOV and the bottom graph shows the resultant instantaneous phase.

$$\phi(x,t) = \int_0^t \gamma \cdot A \cdot e^{-\left(\frac{V\tau - x}{\sigma}\right)^2} d\tau = \gamma \frac{A\sigma\sqrt{\pi}}{V} erfc\left(\frac{Vt - x}{\sigma}\right) \quad (3)$$

$$s(t) = \int_{-\infty}^{+\infty} m(x) \cdot e^{-i\left(\gamma \frac{A\sigma\sqrt{\pi}}{V} erfc\left(\frac{Vt-x}{\sigma}\right)\right)} dx \quad (4)$$

$$s(t) = m(Vt) * e^{-i\left(\gamma \frac{A\sigma\sqrt{\pi}}{V} erfc\left(\frac{Vt}{\sigma}\right)\right)} \quad (5)$$

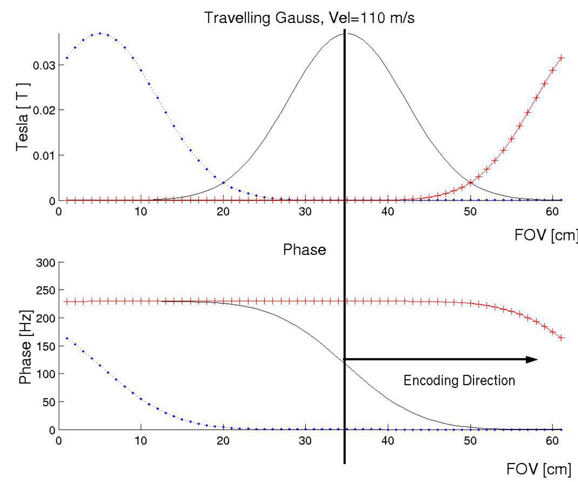

Fig1: Top) Traveling Gaussian Pulse moving from left to right. Bottom) Phase induced on the spins as the wave passes over them

Discussion. Eq. 5 shows that the object is now a function of time, therefore the resolution is proportional to the sampling rate. The greater the velocity in which the pulse crosses the FOV, the fastest the scan. A shortcoming of this technique is the feasibility of creating such a pulse with the current hardware. One way to overcome this problem would be to make the pulse wider, thus with smaller spatial derivatives, lessening the demand on the hardware. On the other hand, it is also possible to adapt the proposed method to existing magnetic field non-linearities, as long as they have the same properties of been spatially limited (to allow the convolution), and traveling (a moving table could be an example). Also, there is a limitation in the practicability of de-convolving the signal. A two dimensional extension could be achieved, using frequency encoding in one direction and the traveling pulse in the other, or as in Projection Reconstruction, encoding with a traveling pulse at different angles. This method can be used with any moving magnetic field of finite in length.

Conclusions. We have proposed and exemplified the use of non-linear magnetic fields to encode an imaged object. This field has to be of finite spatial length and it has to travel across the FOV. The recovery of the image is achieved by deconvolving the acquired signal with the dephasing function, particular to the moving field, directly in the space domain. The resolution is proportional to the field speed and the sampling rate.