A Parallel Image Reconstruction for Real-Time MRI: Neither SMASH nor SENSE

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Introduction

With parallel MRI techniques significant reductions in image acquisition time and, thus, in local SAR can be achieved. Two fundamentally different reconstruction techniques for parallel MRI are SENSE [1] and SMASH [2]. With SENSE, reconstruction is performed in image space using previously acquired coil sensitivity maps. Therefore, SENSE reconstruction process can only be started after the complete raw data are sampled. The SMASH algorithm reconstructs missing k-space data using linear combinations of measured k-space lines so that an interleaved data acquisition and k-space reconstruction are possible. The SMASH reconstruction factors can be determined in pre-calculations [3] or, as with AutoSMASH, directly from additionally acquired k-space lines [4]. In this work a reconstruction algorithm in k-space is presented, which is based on a least squares method which accomplishes the optimal reconstruction from known coil sensitivities.

Theory

With known coil sensitivity maps $C^i(x, y)$ the measured signal $S^i(x, k_y)$ after Fourier transformation in readout direction of the *i*th coil is given by

$$S^i(x,k_y) = \int C^i(x,y)\cdot \rho(x,y)\cdot e^{-ik_y y}\,dy \qquad (1)$$

With the SMASH reconstruction the missing lines $k_y+m\Delta k_y$ can be determined through a weighted superposition in hybrid (x,k_y)-space:

$$S^{Comp}(x,k_y+m\Delta k_y) = \sum_j n_m^j(x) S^j(x,k_y)\ . \qquad (2)$$

To determine the weighting factors n^j_m the coil sensitivities $C_m(x,y)$ are expressed in multiples of the spatial frequency $m\Delta k = 2\pi m/\mathrm{FOV}$

$$C_m(x,y) = \sum_j n_m^j(x) C^j(x,y) = e^{im\Delta k_y y}\ . \qquad (3)$$

Using a least squares approximation an optimal set of weighting factor can be derived:

$$n_m^j(x) = \sum_j [D_{i,j}^{-1}(x) C^i(x,m)]^* \qquad (4a)$$

with

$$D_{i,j}(x) = \sum_{k_y} C^j(x,k_y)[C^i(x,k_y)]^* \qquad (4.b)$$

Note, that only the coil sensitivities are required to compute the weighting factors, so that the factors can be determined prior to the parallel data acquisition. Using the pre-computed factors in Eq. 4a the sum in Eq. 2 can be evaluated for each measured line in k-space directly after the data acquisition.

The final phase of the image reconstruction then consists of a Fourier transform in phase encoding direction at the end of the measurement:

$$\rho(x,y) = \sum_g e^{i(g\Delta k\, y)} S^{Comp}(x,g)\ , \qquad (5)$$

where $g = (m+k)$ mod FOV. Inserting Eqs. 2 and 4 into Eq. 5 finally yields:

$$\rho(x,y) = \sum_j \underbrace{\sum_i D_{i,j}^{-1*}(x) C^i(x,y)^*}_{U^j(x,y)} \underbrace{\sum_{k_y} e^{i(k_y y)} S^j(x,k_y)}_{S^j(x,y)} \qquad (6)$$

which is essentially the formula for SENSE image reconstruction.

Material and Methods

The reconstruction method was implemented using the software package IDL 5.3 (RSI Inc., Boulder, CO). A vessel phantom was imaged at a clinical 1.5T MR scanner (Siemens Symphony, Erlangen, Germany) with 3 elements of the integrated spine array coil. Only half the FOV in phase encoding direction was acquired. The coil sensitivities were determined prior to the measurement in a phantom of homogeneous spin density.

To assess to which extent the weighting factors in Eq. 2 contribute to the final image, a series of images was reconstructed using sub-sets of the weighting factors with increasing size. The squared image distance (i.e. the sum over the squared differences) between the sub-sets and the full reconstruction using all weighting factors was computed as a figure-of-merit for the image reconstruction.

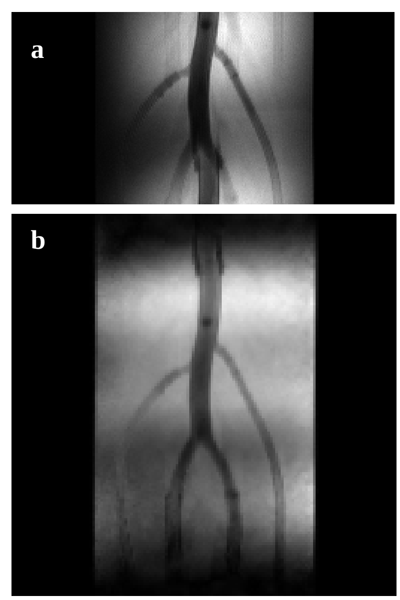

Fig 1.: (a) Conventional sum-of-squares image of the FOV-reduced data set showing the vessel phantom. (b) Full image after reconstruction using optimal weighting factors.

Results and Discussion

Figure 1 compares the standard sum-of-squares reconstruction (a) with the reconstruction using Eq. 5 (b). In the full-FOV data set, no foldover artifacts can be observed and the image quality is comparable to a conventional SENSE reconstruction. In Fig. 2 the squared image distance between a reduced and a full weighting factor reconstruction is plotted. In this example no significant improvement in image quality can be expected for a dimension *m* of $n_m^j(x)$ larger than 5.

The algorithm in Eq. 5 represents a version of SENSE which can be directly used during image acquisition, once the coils sensitivities are known. Compared to SMASH the harmonic expansion of the coil sensitivities is not truncated at the number of receiver coils, so that an analytically optimal image representation can be achieved in the least-squares sense. The algorithm can be applied for realtime reconstructions and can be numerically optimized by using only a reduced dimension of the weighting matrix.

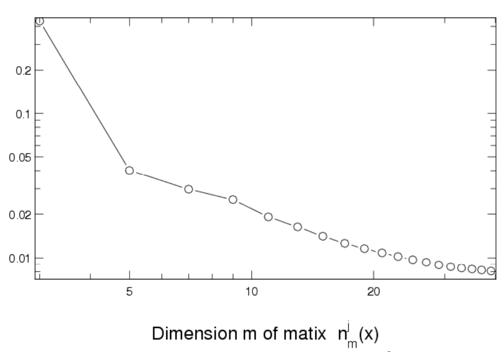

Fig. 2: Squared image distance as a function of the dimension m of the weighting matrix.

References

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