Efficient 3D SPACE-RIP

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Introduction: 3D parallel imaging utilizes phase encoding along two dimensions, which enables rapid image acquisitions by downsampling along both phase encoding dimensions. Reconstruction of the image using 3D Space-RIP [1] and Generalized SENSE [2] requires large computational and memory resources. For example, with a speedup of 4x (using ½ the full encodes along each dimension), reconstruction of a 256-by-256 slice with data from four coils requires a linear system matrix of size 65536 by 65536 (128-128-4 by 256-256). In contrast, the 2D SENSE approach [3] with uniform downsampling along both phase encode directions is not memory intensive. However it is difficult to suppress aliasing artifacts in this method --- even with Tikhonov-style regularization approaches. Here, we demonstrate a method to reduce the computational load of 3D Space-RIP, by efficiently inverting the system matrix formed by phase encoding with irregular downsampling along one direction, and uniform downsampling along the other.

Theory: 3D Space-RIP reconstructs an image by solving the inverse problem $S=\mathbf{P}\rho$. Here, S is the acquired coil data stacked vertically in a vector, and ρ is a vector corresponding to the FOV spin distribution to reconstruct. $\mathbf{P}$ is the system matrix, constructed from estimates of the coil sensitivities and the particular phase encodes used during acquisition. This can be represented compactly as a stack of matrices, each of the form $[(Q^H \otimes P^H)\mathrm{diag}\{\mathrm{vec}\{W_l\}\}]$ where W_l is the coil sensitivity estimate for coil l, and the rows of Q^H and P^H correspond to the Fourier encoding harmonic terms, $exp\{j\gamma(G_r{}^h n\tau)\}$, induced by phase encode h and gradient direction G_r. Q^H and P^H are effectively rows of a Fourier Transform matrix operator with mutually orthogonal rows. That is, $Q^H Q=I_m$ and $P^H P=I_n$, where m and n are the number of phase encodes employed along the first and second phase encoding dimensions, respectively.

Posing the parallel coil image reconstruction problem as a system of normal equations, $\mathbf{P}^H S=\mathbf{P}^H\mathbf{P}\rho$, exposes the point-spread-function (psf) imposed by the down-sampling choice. In the case of uniform down-sampling along both phase encode directions, this psf is a weighted aliasing operator:

$\mathbf{P}^H\mathbf{P} = [((\mathbf{1}_\mathrm{f} \otimes I_{\mathrm{m/g}}) \otimes (\mathbf{1}_\mathrm{g} \otimes I_{\mathrm{n/g}})) \circ (\sum_{l=1}^{L} \mathrm{vec}\{W_l\}\ \mathrm{vec}\{W_l\}^\mathrm{H})\ ]$ whose sparse structure is shown in Figure 1. It is this structure that allows minimal computational load in 2D-SENSE [3] implementations.

This uniform downsampling sparseness in the parallel imaging normal equations can be used to decrease the memory requirements of 3D Space-RIP as well. In particular, using irregular down-sampling along one phase encode direction and uniform down-sampling in the other, one can construct a similar sparse system matrix. This particular sampling pattern creates a sparse normal system matrix that contains multiple non-zero blocks, of the form $(\mathbf{1}_\mathrm{f} \otimes I_{\mathrm{m/g}}) \otimes (\mathbf{1}_\mathrm{g} \otimes PP^H)$. The psuedo-inverse of each non-zero block can be found independently and used collectively to solve the Space-RIP linear system.

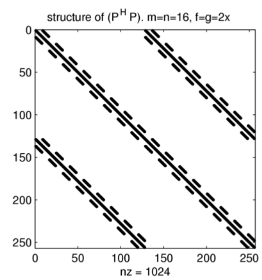

Figure 1: normal equation matrix sparsity pattern with 2x—by-2x acceleration

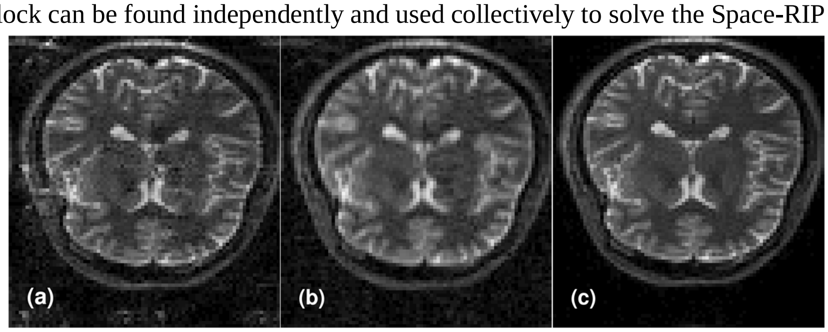

Figure 2: Recon using (a) 2D-Sense, (b) Space-RIP, (c) no acceleration

Methods and Results: Full 3D volume four-coil data was acquired on at 1.5T MR scanner using a high-resolution 3D Fast Spin Echo sequence, with coils arranged circumferentially around the volunteers head. Two sampling patterns were used: one with uniform downsampling by 2 along both phase encode directions, and one with an irregular downsampling along one direction and uniform downsampling along the other. The irregular sampling pattern employed a phase encode selection weighted by the density of energy in k-space. One could alternatively use the optimal scheme described in [4]. The acquisition speed-up factor (4x) was equivalent in both cases. Fig. 2(a) shows the reconstruction using a 2D-SENSE reconstruction, while Fig. 2(b) shows the efficient 3D Space-RIP reconstruction. For reference, a full resolution image is shown in Fig. 2(c). It is notable that the use of irregular sampling signifiantly reduces the appearance of aliasing artifacts, even though the regularization parameters used in the two reconstructions were equivalent. On a SUN Ultra-80 with 4GB RAM, the computation time to reconstruct the Space-RIP image (74.7 sec) was longer than the 2D-SENSE reconstruction (28.1 sec), but significantly shorter than the time to invert the full Space-RIP matrix (7228.7 sec = 2 hours).

Summary: This approach to Space-RIP reconstructions provides an excellent balance between artifact suppression, the ability to adapt the phase encoding pattern to a meet a particular need, and fast image reconstruction time. We expect this will enable rapid clinical implementation of 3D parallel acquisitions using readily available hardware.

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