Using a Least-Square Sense Error Minimization Approach in the Determination of Ferric Ion Diffusion Coefficient in MRI-Fricke-Infused Dosimeter Gels

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ABSTRACT

A numerical method was adopted to solve the diffusion equation to determine ferric ion diffusion coefficient in Fricke-agarose gels. A fast MR acquisition technique was employed to avoid smearing of acquired data due to diffusion over an extended time period. Our results show that for a Fricke gel contained 1mM ammonium ferrous sulfate, 1% agarose, 1mM sodium chloride and 50mM sulfuric acid, its ferric ion diffusion coefficient is 1.73×10^{-2}cm^{2}h^{-1} in room temperature. This value is consistent with the $1 \sim 2\times10^{-2}$ cm^{2}h^{-1} range obtained by previous studies under varying concentrations of gel ingredients.

INTRODUCTION

Ferric ion diffusion is a detrimental factor in MRI-Fricke infused gel dosimetry [1, 2]. It is important to have an accurate measure of ferric ion diffusion coefficient in order to understand its effects in radiation dosimetry. Based on a set of MR images acquired with different ferric ion diffusion delay periods, a least square post-processing method was used to solve the diffusion equation and calculate the ferric ion diffusion coefficient. The corresponding dose distribution profile evolutions can be recorded from these MR image intensity changes [3]. The employment of the image-based dosimetry approach reduced the MR acquisition time by several folds compared to the conventional R1-based methods [1], thus suffered much less smearing of the dose profiles and enables a more effectual way to calculate ferric ion diffusion coefficient. The calculated diffusion coefficient was adopted to simulate the ferric ion diffusion phenomenon where the after-effect caused by the ferric ion diffusion in MR dosimetry experiment was compared for the consistency.

METHODS & MATERIALS

MR image-based differential gel dosimetry method [3] was used to record ferric iron distribution after varying elapsed diffusion times. This ferric ion distribution was then transformed into absorbed dose profiles for further processing. A total of nine images were taken with the parameters: TR/TE: 500/11ms, slice thickness: 3mm, FOV: 26cm, matrix size: 256x256, and NEX: 2. The first image is taken before Gamma Knife irradiated the gel phantom. The second is taken immediately after radiation. The rest seven images were taken at a delay time of 30, 45, 60, 75, 90, 105 and 120 minutes after the second image. A total of eight differential MR images were produced by subtracting images acquired after irradiation from the first image. These are denoted as ΔS maps.

The diffusion phenomenon is governed by the following equation,

$$D\nabla^2\rho = \frac{\partial\rho}{\partial t} \quad \Lambda\Lambda \quad (1)$$

where D is the diffusion coefficient and ρ is the concentration distribution. This diffusion equation could be simplified using the explicit method, which replaces the derivative terms in differential equations with difference equations. Thus

$$\frac{\hat{\partial}\rho}{\partial t} \equiv \frac{\rho(x_i,y_i,z_i;t_{j+1}) - \rho(x_i,y_i,z_i;t_j)}{\Delta t} \quad \Lambda\Lambda \quad (2a) \qquad \frac{\hat{\partial}^2\rho}{\partial x^2} \equiv \frac{\rho(x_{i+1},y_i,z_i;t_j) - 2\rho(x_i,y_i,z_i;t_j) + \rho(x_{i-1},y_i,z_i;t_j)}{(\Delta x)^2} \quad \Lambda\Lambda \quad (2b)$$

$$\tilde{\rho}(x_i,y_i,z_i;t_{j+1}) = \rho(x_i,y_i,z_i;t_j) + \Delta t \times D \times \left(\frac{\hat{\partial}^2\rho}{\partial x^2} + \frac{\hat{\partial}^2\rho}{\partial y^2} + \frac{\hat{\partial}^2\rho}{\partial z^2} \right) \quad \Lambda\Lambda \quad (2c)$$

The diffusion coefficient, D, was iteratively assigned in Eq. (2c) and a least square sense error minimization approach was adapted to determine the optimal D when minimum error emerged between the difference of the predicted distribution and empirical value extracted from MR images.

RESULTS

Figure 1(a) demonstrated the MR image intensity change (ΔS map) due to irradiation, while Fig. 1(b) are the difference maps between the first image and the rest seven images obtained at varying delay times of 30, 45, 60, 75, 90, 105, and 120 minutes, respectively. Table I and Fig. 2 indicate that the minimum normalized standard error emerged at D=1.73x10^{-2} cm^{2}h^{-1}. Figure 3(a) is the simulated result of the MR image intensity change (ΔS map), while Fig. 3(b) are the difference maps generated using the derived diffusion coefficient.

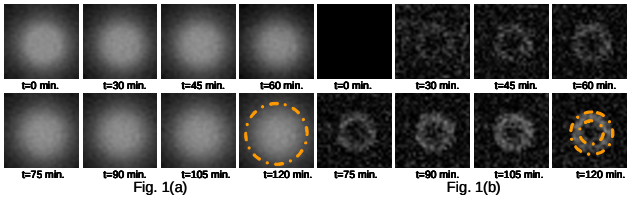

Fig. 1(a) Fig. 1(b)

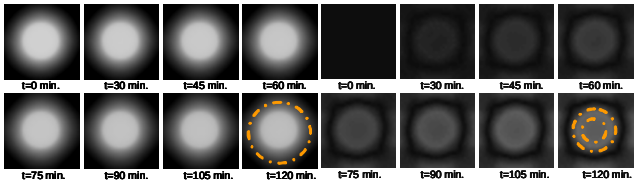

Fig. 3(a) Fig. 3(b)

Table I	
Diffusion coefficient D (cm^{2}h^{-1})	Normalized Standard Error
1.65×10^{-2}	142.65103407
1.67×10^{-2}	142.64846662
1.69×10^{-2}	142.64667345
1.71×10^{-2}	142.64565454
1.73×10^{-2}	142.64540990
1.75×10^{-2}	142.64593953
1.77×10^{-2}	142.64724343
1.79×10^{-2}	142.64932160

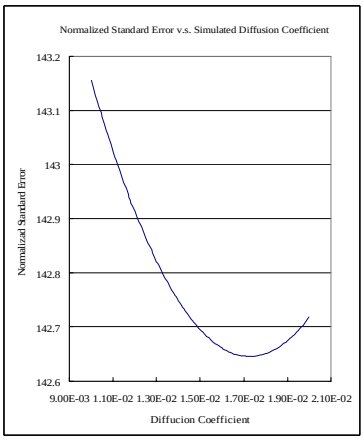

Fig. 2

DISCUSSION & CONCLUSIONS

A fast method was introduced to calculated ferric ion diffusion coefficient in MRI-Fricke gel dosimetry. The obtained diffusion coefficient coincides within the $1 \sim 2\times10^{-2}$ cm^{2}h^{-1} range reported by previous studies under varying concentrations of gel ingredients [1, 2]. The ring-shaped artifact and its size (Fig. 3) are in close agreement to those derived from MR empirical data.

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