Quantification of Field Changes due to Tissue Susceptibility Variations

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Introduction

Quantification of susceptibilities is an important subject in MRI. MR phase images can be used to retrieve susceptibilities of tissues. One application is to obtain the blood oxygenation level through the measurement of phases from images (e.g., [1]). Recently, reference [2] describes how one can quantify magnetic susceptibilities for arbitrarily shaped objects in any fields with minimal inhomogeneity. In this abstract, we will summarize our theory development as based on fundamental electromagnetism and reevaluate some of the results shown in [2].

Theory and Methods

An external magnetic field induces field corrections due to the body and tissue susceptibilities depending on their geometries. Assuming that there is no eddy current induced in the subjects during imaging, the induced fields can be expressed by the spatial derivative of the solution of Laplace's equation [3]. Because the susceptibility of any tissue is much smaller than unity, only the first-order term in the expansion of the susceptibility makes a major contribution to the solution. As a result, the *H*-field component along the main field B_0 is approximated by $(1 - \chi\, \alpha(\vec{x}))\, B_0/\mu_0$ where χ is the susceptibility of the tissue, μ_0 is the permeability, and $\alpha(\vec{x}) = \frac{1}{4\pi} \oint d\vec{S}' \bullet \hat{B}_0 (\hat{B}_0 \bullet \vec{\nabla} (1/|\vec{x} - \vec{x}'|))$. The integral is over the surface of the tissue. Inside the tissue, the magnetic field component along the main field is $\mu_0 (1 + \chi)\, H \approx (1 + \chi - \chi\, \alpha(\vec{x}))\, B_0$. Outside the tissue, assuming an empty space, the magnetic component along the main field is $\mu_0 H \approx (1 - \chi\, \alpha(\vec{x}))\, B_0$. The field difference picked up by MRI is $\Delta B = (1/3 - \alpha(\vec{x}))\, \chi B_0$ or $-\chi\, \alpha(\vec{x})\, B_0$, depending on whether point $\vec{x}$ is inside or outside the tissue, respectively.

When more tissues are present in one system, more complicated formulas can still be written down, based on these equations of field differences. For example, if a tissue with susceptibility χ_1 is inside another tissue with susceptibility χ_2, the field difference of a point $\vec{x}$ inside the former tissue is $\Delta B = (1/3 - \alpha_1(\vec{x}))\, \chi_1 B_0 - (\alpha_2(\vec{x}) - \alpha_1(\vec{x}))\, \chi_2 B_0$ where $\alpha_1(\vec{x})$ and $\alpha_2(\vec{x})$ are the integrals over the surface between the first tissue and the second tissue, as well as over the outer surface of the second tissue, respectively. The phase in MR images is the field difference multiplied by $-\gamma$ (gyromagnetic ratio times 2π) and the echo time.

Results and Discussions

The formulas of $\alpha(\vec{x})$ can be analytically derived when simple geometries are considered. As an example, for a sphere with radius *a*, $\alpha(\vec{x}) = 1/3$ or $a^3 (1 - 3\cos^2\theta)/(3r^3)$, when $\vec{x}$ is inside or outside the sphere, respectively. θ and *r* are spherical coordinates. For an infinitely long cylinder with radius *a* and whose axis is at an angle θ_0 with the main field, $\alpha(\vec{x}) = (\sin^2\theta_0)/2$ or $-a^2 (\sin^2\theta_0 \cos 2\varphi)/(2\rho^2)$, when $\vec{x}$ is inside or outside the cylinder, respectively. ρ and φ are cylindrical coordinates. These are well-known results often used by MR researchers [1,2].

Reference [2] employs a spherical-mean-value theorem in order to carry out a derivation. This translates to an average of $\alpha(\vec{x})$ over a spherical surface with radius $|\vec{x}|$ (call it $\bar{\alpha}$). Now it becomes clear and it can be easily shown that $\bar{\alpha} = \alpha(0)$ (the value of α at the sphere center) when the spherical surface is smaller than the object in which it has a uniform susceptibility. When the spherical surface is larger than the object with an arbitrarily shape, $\bar{\alpha} = 0$. It is obvious that this average can be applied to a sizable spherical shell or a sphere. The averaging results will still be the same. In fact, the MR experimental data shown in Fig. 4 of [2] support the theory and imply constant α values inside the pillboxes and different constant α values outside the pillboxes (but inside the test beaker).

Based on the experimental results shown in Fig. 4 of [2], the theoretical prediction of the susceptibility difference between the solutions inside and outside a cylindrical pillbox should be 0.981 ppm instead of 0.921 ppm, when the cylindrical pillbox is parallel to the main field. The discrepancy is due to a certain confusion of how data are to be analyzed and how the susceptibilities are to be averaged in [2]. Our derivation here also leads to a result that the susceptibility of water (doped with a small amount of $CuSO_4$) used in the beaker in [2] is about -8.22 ppm, in good agreement with the known susceptibility of water, -9.05 ppm.

In summary, the first-order estimate of susceptibilities of tissues only depends on geometries of tissues. This may become a powerful tool when susceptibility-weighted imaging becomes more important [4].

References

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