# Non-linear Phase Correction with an Extended Statistical Algorithm


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## Introduction

In the past few years, many phase correction methods have been proposed. One example is the statistical linear phase correction method proposed by Ahn and Cho (AC method)[1]. The AC method is efficient and robust, and its Fourier counterpart has recently been found [3]. However, we found that its straightforward extension to higher orders was challenged by reduced signal-to-noise ratio (SNR) due to 1-pixel-shift differential phase calculation. In this work, we introduced n-pixel-shift rotational differential field (RDF) that represents local vector rotations of a complex image relative to itself after being shifted by n pixels. We have found that a larger shift offers a remarkable SNR improvement, successfully extending the AC method to handle the $2^{nd}$ order non-linear phase errors. The n-pixel-shift RDF can also be applied to improve other methods such as the weighted least square phase unwrapping method proposed by Liang (WLS method)[2].

## Methods

### Theory

The AC method statistically fits a linear polynomial to phase variations in MRI. Given a 1D complex field denoted as C(x), the linear coefficient is determined by the phase of the statistical expectation of the complex field given by $C(x)C^*(x-1)$ [1], which we call the 1-pixel-shift RDF of C(x). The feasibily and robustness of the AC method are widely appreciated, and its Fourier counterpart can be directly derived using the Fourier power theorem as follows: $\sum C(x)C^*(x-1) = \sum |F(k_x)|^2 \exp(i2\pi\, k_X/N)$ [4]. The Fourier counterpart of the AC method is intuitively interpreted in Fig. 1, which shows an angular distribution of $|F(k_x)|^2\exp(i2\pi\, k_x/N)$[3]. In Fig. 1, the gray circle represents noise, and the dashed vectors represent signal peaks. The angle of the spectrum centroid denoted by the bold vector gives the linear coefficient. However, the AC method fails to handle higher order non-linear phase errors because the SNR is greatly reduced by the 1-pixel-shift rotations.

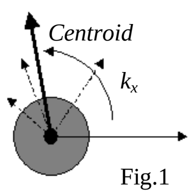


Fig.1

In this work we developed the n-pixel-shift RDF that represents local vector rotations of a complex field relative to itself after being shifted by n pixels. Consider a 1D case, the *n*-pixel-shift RDF (denoted as $\Delta n_x$) of C(x) is expressed as $C(x)C^*(x-n)$. The amount of pixel shift was found to be crucial since a larger shift greatly enhances the SNR in the estimated phase terms, allowing a much more accurate phase correction. The reasons are illustrated in both image space and k-space as follows: In image space, if a 1D noisy complex image C(x) is given as the addition of signal S(x) and Gaussian noise N(x), $\Delta n_x = C(x)C^*(x-n) = S(x)S^*(x-n) + N(x)S^*(x-n) + S(x)N^*(x-n) + N(x)N^*(x-n)$. When a summation is performed over the entire field, the contribution from the last 3 terms should not depend on pixel shift "*n*", unlike that from the first term that has a larger phase angle when *n* is bigger. In k-space, the Fourier counterpart of $C(x)C^*(x-n)$ is given by $|F(k_X)|^2\exp(i2\pi\, n\, k_X/N)$. Fig. 1 shows that the shift *n* should not change noise distribution due to phase wrapping, but increases the phase angle of the centroid by a factor of *n* as long as it is not wrapped.

Based on the n-pixel-shift RDF, we extended the AC method to perform the $2^{nd}$ order phase correction. It is divided into 3 steps: (1) A proper total shift size is chosen. This is because the advantages of a larger shift are counterbalanced by the greater loss of signal points near tissue boundaries and the higher likelihood of phase wrapping in the RDF. (2) The $1^{st}$ and $2^{nd}$ order RDFs are calculated with the half total shift size, yielding the greatest SNR amplifying factor given a total shift size. (3) Phase calculation and correction begin from the $2^{nd}$ order to lower order until $0^{th}$ order phase correction is finished.

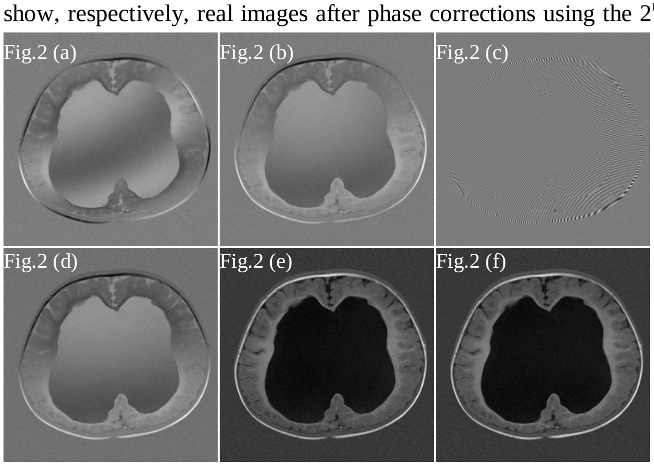
The n-pixel-shift RDF can also be applied to improve the WLS method proposed by Liang [2], which fits a polynomial to phase derivative fields. Here we fit a polynomial to the $1^{st}$ RDFs because even 1-pixel-shift RDFs should be practically advantageous over the phase derivative fields.

### Experiments

The performance of various phase correction methods was evaluated based on existing patient data from a clinical 1.5 T scanner with an Inversion-Recovery (IR) Spin-Echo sequence (TR/TI/TE: 2000/600/30 ms).

## Results

Fig. 2(a) is the real part of the original complex image from a transverse head IR scan. Fig. 2(b) is the real image after phase correction by the linear AC method based on traditional 1-pixel-shift RDF. Fig. 2(c) is the real image after phase correction by the $2^{nd}$ order AC method based on traditional 1-pixel-shift RDF, which gives biased polynomial coefficients by noise and yields a worse result with extremely rapid phase change across. Fig. 2(d) is the real image after phase correction by traditional $2^{nd}$ order WLS method based on the 1-pixel-shift RDF. Figs. 2(e) and (f) show, respectively, real images after phase corrections using the $2^{nd}$ order extended AC method and the $2^{nd}$ order improved WLS method based on the new 10-pixel shift RDF. The improvements shown in the figures can be consistently quantified by a refocusing factor that is the total magnetization in the filed of view (FOV).




## Conclusions

We have demonstrated that phase correction can be improved by SNR enhancement as a result of the n-pixel-shift RDF. The AC method has been extended to handle non-linear terms, and the WLS method has been improved as well, both by using the n-pixel-shift RDF. The same principle can be used in applications involving higher terms and dimensions.

## References


[1] C. B. Ahn and Z.H. Cho, IEEE Trans Med Imag 1987; 1:32-36. [2] Z.P. Liang, IEEE Trans Med Imag 1996; 15:893–897. [3] J. A. Derbyshire, et al., 7th ISMRM, p.2002,1999. [4] R.N. Bracewell, The Fourier transform and its application, New York, NY, McGraw-Hill.