A GENERAL FORMALISM FOR DIFFUSION AND TURBULENT FLOW IMAGING

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Introduction

Motion encoding has important applications in MRI. In flow quantification, coherent motion is encoded although incoherent motion (turbulence) can be present. The best-known application of incoherent motion encoding is (of course) diffusion MRI.

The encoding of motion using magnetic field gradients can be analyzed using a linear systems approach (1,2). We shall combine this approach with the phase space (i.e. position-velocity space) concept (3). The formalism will be explained on the basis of two variants of the same basic sequence: the Fourier Flow Method (FFM in flow imaging) (4,5) and the Pulsed Gradient Spin Echo (PGSE in diffusion imaging) (6). The formalism allows us to interpret quantities such as diffusion time and measured displacement profiles when doing q-space imaging (6) with general gradient waveforms.

Pulse sequences

The PGSE-sequence has two positive gradient lobes, one on each side of the inversion pulse as shown in Fig.1. The dephasing of the spins can be obtained by introducing an effective gradient which is exactly the gradient waveform used in the FFM-sequence (usually in a gradient echo version).

Linear systems approach

The motion induced phase shift acquired by spins that move along a magnetic field gradient can be considered as a function of the time t the gradient is switched off:

$$\varphi(t) = \gamma \int_{t-T}^{t} g(t')z(t')dt' = h_0(t) \otimes z(t) \qquad \text{with} \qquad h_0(t) = \gamma g(T-t) \qquad [1]$$

Here $g(t')$ is the gradient waveform, T its duration and $z(t')$ the time dependent position of the spins. Equation [1] shows that position encoding can be considered as a linear system (convolution relation) with impulse response $h_0(t)$. This is valid when considering one gradient lobe. When considering the whole waveform (two lobes), because the zero-th gradient moment $m_0 = 0$, not position but velocity is encoded:

$$\varphi(t) = h_1(t) \otimes v(t) \qquad \text{with} \qquad h_1(t) = \int_0^t h_0(\tau)d\tau \qquad [2]$$

This can also be interpreted in terms of a transfer function $H_1(\omega)$ by taking the Fourier transform: $\Phi(\omega) = H_1(\omega)V(\omega)$. It is clear that the transfer function depends on the spectrum of the gradient. According to the linear systems approach, flow encoding should be interpreted as a weighted averaging process with weight function the impulse response. When interpreting it as instantaneous encoding, the centroid of the impulse response is the best instant to choose (2). For diffusion imaging in terms of position encoding, this leads to position encoding at the centers of the two gradient lobes in Fig.1. As a result, this leads to a diffusion time Δ (and not $\Delta - \delta/3$) because $\varphi = \gamma m_0 [z(t_{c2}) - z(t_{c1})]$ with $m_0 = g\delta$ and $\Delta = t_{c2} - t_{c1}$. In terms of velocity encoding, the centroid is located halfway the two gradient lobes. The phase shift is then given by $\varphi = \gamma m_1 v(t_c)$ which is completely equivalent to the previous result because $m_1 = g\delta\Delta$ and the velocity is best approximated by $v(t_c) = [z(t_{c2}) - z(t_{c1})]/\Delta$.

Imaging in phase space

Flow imaging can be described in (6D) phase space by using a position-velocity dependent spin density $\rho(r,v)$. The MR-signal becomes then:

$$S(k,q) = \iint \rho(r,v)\exp[i(k.r + q.v)]drdv \qquad [3]$$

with k the zero-th moment of the position encoding gradients and q the first moment of the velocity encoding gradients.

Turbulent flow can be decomposed into its (ensemble) averaged and random components: $v = U + u$. We shall consider FFM with velocity encoding v along the vessel and position encoding x perpendicular to it. For a general gradient waveform, this leads in the Gaussian Phase Approximation (GPA) (7) to an image intensity $I(x,v) = I_1(x,U) \otimes I_2(u)$ with:

$$I_2(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-u^2}{2\sigma^2}\right) \qquad \text{and} \qquad \sigma^2 = \frac{1}{2\pi}\int |H_{1n}(\omega)|^2 R_u(\omega)d\omega \qquad [4]$$

In q-space imaging of diffusion, two spatial encodings are used and $U = 0$. This leads to a measured velocity distribution $I_2(u)$ for each pixel (x,y). The variance σ^2 depends on the normalized (w.r.t. $\omega = 0$) transfer function and the Fourier transform of the autocorrelation function (diffusion spectrum). Equation [4] shows that $\sigma^2 < \langle u^2 \rangle$ due to the gradient spectrum. As a result, measured distributions underestimate the width of the real distribution when using finite gradients. This was for instance experimentally observed in ref. (8). The formalism can also be applied for other sequences (e.g. phase mapping with turbulent pulsatile flow). For restricted diffusion, higher order terms should be included in the GPA.

References

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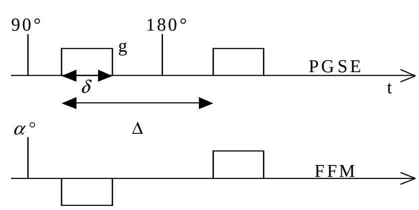

Figure 1