3D Rotation Matching Using Spherical Harmonic Transformation of k-Space Navigator

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Introduction:

It has been demonstrated [1] that rigid 3D rotation can be detected by a short navigator pulse that samples a sphere in 3D k-space. The current technique of computing the 3D rotation employs an exhaustive search by using least square minimization. It is time consuming (a few seconds) and also requires identical coverage of the sphere from all navigator pulses during the scan. Cross-correlation has been used [2] to identify 2D rotation from orbital navigators by taking advantage of FFT acceleration. The aim is to find the angle that maximizes the rotational correlation function (RCF). The cross-correlation of two spheres can be computed efficiently with FFTs if the spherical data are expressed in spherical harmonics. This method has many applications in molecular biology, cosmology and geography, but has never been utilized in MRI. Here I report the preliminary result of the 3D rotation matching using FFT-accelerated cross-correlation in simulated k-space data, as an evaluation of its potential for future applications in MRI.

Method:

(1) $f(\mathbf{u}) = \sum_{l=0}^{bw-1}\sum_{m=-l}^{l} f_{lm}Y_{lm}(\mathbf{u}),\ g(\mathbf{u}) = \sum_{l'=0}^{bw-1}\sum_{m'=-l'}^{l'} g_{l'm'}Y_{l'm'}(\mathbf{u})$

(2) $R[g(\mathbf{u})] = g[R^{-1}(\mathbf{u})] = \sum_{l'=0}^{bw-1}\sum_{m'=-l'}^{l'} g_{l'm'}Y_{l'm'}(R^{-1}\mathbf{u})$

(3) $Y_{l'm'}(R^{-1}\mathbf{u}) = \sum_{n} D^{l'}_{nm'}(R)Y_{l'n}(\mathbf{u})$

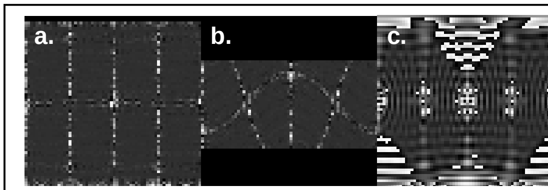
(4) $D^{l}_{nm'}(\alpha,\beta,\gamma) = \exp(-in\alpha)d^{l}_{nm'}(\beta)\exp(-im'\gamma)$

(5) $C_R = \int_{\rho} f\cdot\overline{R[g]}\,d\rho^3$ 6) $d^{l}_{nm'} = \sum \omega\exp(-ih\beta)$

(7) $C_R = \sum_{l,m,m'} \overline{\omega} f_{lm}\overline{g}_{lm'}\exp(in\alpha + ih\beta + im'\gamma)$

Theory Let f and g be the finite-size reference map and detection map, respectively, sampled on the surface of a sphere in spherical coordinates ($\mathbf{u}$). Both functions can be expanded (Eq.1) in series of spherical harmonics (Y_{lm}) since they form an orthonormal basis for the spherical coordinates. Here, f_{lm} and $g_{l'm'}$ are the spherical harmonic coefficients and bw is the bandwidth of the sample. For a rotation R on g, we have Eq.2. Under rotation, Y_{lm} can be expressed as Eq.3 in terms of Wigner D-functions (D^{l}_{mn}). Any 3D rotation can be expressed by Euler angles. Here we use the 'ZYZ' convention, i.e. $R(\alpha,\beta,\gamma)$ means rotate by γ about the original Z axis, then by β about the original Y axis, and finally by α about the original Z axis. The D^{l}_{mn} is related to Euler angles by Eq.4. To find the rotation $R(\alpha,\beta,\gamma)$ on g that maximizes its overlap with f, we look for the maximum of the RCF, C_R, as expressed in Eq.5. Using the orthogonality of the Y_{lm}'s, C_R is simplified to Eq.6. From Eq.4 and 6, the Fourier kernel is explicit for two (α and γ) of the three Euler angles. The angle β is embedded in $d^{l}_{nm'}$ of Eq. 4. The challenge of utilizing a full FFT to compute all three angles resides in decomposing $d^{l}_{nm'}$ to a Fourier series. Here a method of Discrete Wigner Transform [3] (DWT) is employed to compute $d^{l}_{nm'}(\beta)$ (Eq.6) so that FFT can be applied to solve for β. In the computation procedure, the term ($\overline{\omega} f_{lm}\,\overline{g}_{lm'}$) is first calculated and then an inverse FFT is employed to compute C_R in (α,β,γ) space. The location of the maximum peak in C_R solves for $R(\alpha,\beta,\gamma)$.

Simulation In order to eliminate errors from interpolation, A 3D object composed of a slab and a sphere was simulated so that the data in k-space can be calculated analytically. The data were obtained on 64×64 latitude and longitude grids on a sphere. While there are many variables to test, here I report the preliminary findings in matching the full spherical data with a subset of data.



Result:

The simulated k-space data are illustrated in the figure: a) reference map, b) detection map, and c) rotation corrected map. A rotation of (α,β,γ) = (0, 30°, 0) was applied to map a) to obtain map b). The correlation function of a) and b) yields a max peak at (α,β,γ)=(0, 29.9°, 0), indicating that a rotation of 29.9° about Y-axis is needed to rotate map b) back to the reference space. To complete the demonstration, I then rotated map b) by 29.9° and obtained map c). The vertical lines in c) show that the rotation was done correctly. The processing time for this sample size is typically 0.1s (3GHz Xeon running Linux), significantly faster than the processing time of a few seconds for exhaustive search.

This technique is capable of matching a full spherical data (Fig. a) with a partial data sampled on the same (but rotated) sphere (Fig. b). Partial data are generated from the detection map by replacing data on some latitude 'orbits' by zero. The number of zero orbits is increased systematically and each time, equal number of orbits in the north and south poles are zeroed. The accuracy of rotation detection is retained when matching the reference map with the partial data of up to 33% of the orbits replaced by zero (as shown in figures). Both rotations along the orbit (about the Z axis) and across the orbit (about the Y axis) yielded similar results.

Discussion and Conclusion:

It is presented here that the FFT-accelerated correlation technique can be used in the detection of 3D rotations in k-space. It is capable of matching a reference data with a partial data, implying time saving in navigator acquisition. Arbitrary trajectories on the surface of a sphere can be handled by first gridding the data onto the spherical grids, which can then be processed with this technique. This method will be further tested with finer sampling in noise simulation as well as in MRI data in vivo.

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3. Kostelec PJ and Rockmore DN, Santa Fe Institute Working Papers Series Paper #03-11-060, 2003.