Noise Analysis in the Multiple Receiver MR Experiment

A. M. Abduljalil[1]

[1]Ohio State University, Columbus, Ohio, United States

Introduction

Quadrature detection of the MR signal is commonly implemented generating a complex set of data points. Images or spectra are extracted after the application of one dimensional or more DFT transformations depending on the dimensionality of the MR experiment. Random noise is digitized with the desired signal. The noise follows a zero centered Gaussian distribution for each component of the noise, both real and imaginary, and can be characterized by its standard deviation σ. In addition, the magnitude of the noise follows the Rayleigh distribution (1). When multiple receivers are used for an imaging experiment, the data are combined typically by calculating the square root of the sum of squares of the individual images. Not only is this method not optimal, but it also overestimates the signal-to-noise-ratio (SNR) (2).

Theory

Magnitude images are normally generated by the pixel by pixel evaluation of the square root of the sum of squares (RSS) of the real and imaginary parts. If more than one receiver is utilized as in the case of phased array experiment; then the measured signal M_n for n receivers can be obtained as $M_n = \sqrt{\sum_{k=1}^{n}\left(S_{rk}^2 + S_{ik}^2\right)}$, where S_{rk} and S_{ik} represent the real and imaginary components of measured signal for each receiver k and for each pixel. In this case, S_k includes the signal A_k and the noise N_k such as for each receiver k, the acquired signal is $S_k = A_k + N_k$. In general A_k can also be complex since there are phase shifts due to the chemical composition of the imaged object, susceptibility induced phase shifts, or non-homogenous B_0. The phase shifts incurred by the object's chemical shifts and micro-susceptibility are useful and need to be preserved in many instances in contrast to the other phase shifts that need to be removed. Generally, the phase of the signal S_k is noncoherent between the signals acquired by the different receivers. Moreover, different pixels in the same receiver may have additional phase component that is not related to the phase in A_k. The RSS method of image combination overcomes these problems creating the magnitude image. However, this method does not enable the calculation of the phase image if it is desired. More importantly, there are SNR implications that cannot be avoided. And therefore, a better solution is to filter out the undesired phase components. Then, the complex sum of the signal from the different receivers will add coherently the A_k component while the combination of N_k remains noncoherent. Consequently the magnitude image can be expressed as $M_n = \sqrt{\left(\sum_{k=1}^{n} S_{rk}\right)^2 + \left(\sum_{k=1}^{n} S_{ik}\right)^2}$ and will be referred to as the SUM case. There are several techniques that can be used to phase the images from the different receivers such as using iterative methods (2), high-pass filter, or even using parallel imaging method like SENSE.

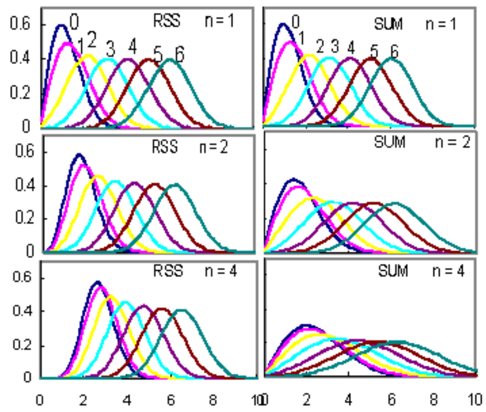
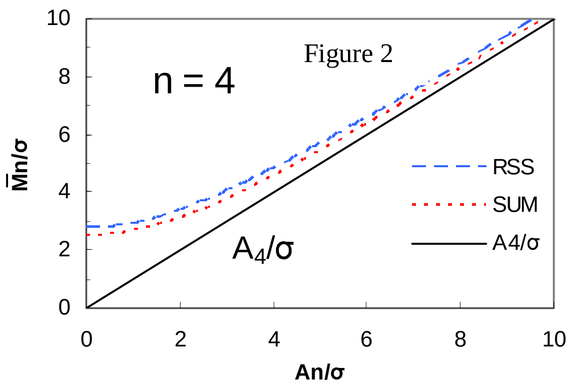
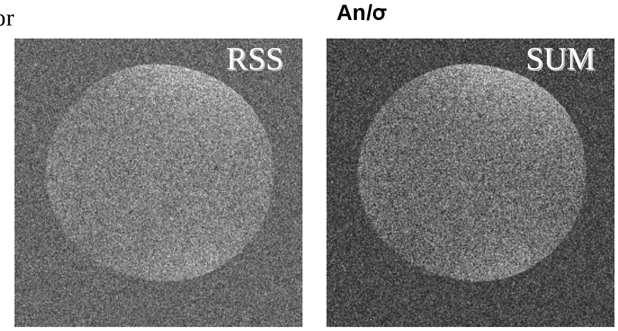


Figure 1.





Method

To evaluate the differences between these two methods, a statistical model was developed using Yorick interpreted language (3). 2048×2048 complex matrices for each receiver were calculated for the Gaussian noise for each receiver. Values of A_k varied and the corresponding $\overline{M}_n$ was calculated. It is assumed that the coupling between the receivers is negligible which is a valid assumption for a well designed phased array coils. In Fig. 1, the left side shows the probability distribution function (PDF) in the RSS case for A_n/σ values of 0-6 and for 1, 2, and 4 receivers. This result is in agreement with the closed form relation (4) $P(M_n) = \left(\frac{M_n}{A_n}\right)^n e^{-\frac{A_n^2+M_n^2}{2\sigma^2}} I_{n-1}\left(\frac{M_n A_n}{\sigma^2}\right)$, where I_{n-1} is the modified Bessel function of the first kind with order of $(n-1)$. The right side of Fig. 1 shows the corresponding SUM case. Fig. 2 shows the calculation of the SNR as a function of A_n/σ for the two cases using four receivers. Clearly, both cases overestimate the SNR, but the RSS values are higher. MR images were acquired showing agreement with Fig. 2 calculations. In addition, the acquired images show a significant improvement in image visualization and in resolving the image from the noise for the SUM case in comparison with the RSS images, particularly for SNR values smaller than 10. The displayed images are mineral oil phantom images acquired using 4 receivers. The measured SNR values are 5.0±1.2 and 3.6±1.4 respectively. The SUM image shows better edge definition and improved detectability.

Conclusion

The sum of images from multiple receivers after correcting for the phase differences shows superior magnitude images in comparison to the square root of the sum of squares case while preserving the phase information.

References

1. Gudbjartsson H, Patz S. MRM 1995; 34:910-914. **2**. Bernstein MA, Thomasson DM, Perman WH. Med Phys 1989;813-817. **3**. Munro DH. Comp Phys 1995; 9:609–615. **4**. Constantinides CD, Atlalar E, McVeigh ER. MRM 1997; 38:852-857.