

Fast Direct Reconstruction of Non-Cartesian k-space Data Using a Combined Discrete and Fast Fourier Transforms

Y. Qian¹, Z. Zhang¹, Y. Wang¹

¹Radiology, University of Pittsburgh, Pittsburgh, PA, United States

Introduction

Non-Cartesian k-space trajectories, such as spirals, are widely used in fast MRI and parallel imaging acquisitions. The standard image reconstruction method is the gridding algorithm that interpolates sampled data onto a Cartesian grid and subsequently uses fast Fourier transform (FT) (1). It provides high precision and fast speed for image reconstruction because of the use of the fast FT. However, the gridding method is strategically not efficient when it is applied to a partial k-space data set, because the fast FT computation for a partial k-space data set is the same as for a full k-space data set (2). For dynamic MRI where interleaved-by-interleaved updating is useful (3), it is desirable to have an efficient method for reconstructing a partial k-space data set.

Methods and Materials

Our fast direct reconstruction (FDR) algorithm consisted of first discrete FT in one direction and then fast FT in the second direction. The discrete FT was chosen to improve accuracy over FFT (2) and was implemented with the equal-phase-line method that accelerated discrete FT by 2.5 times (4). For the implementation of fast FT, a nearest-neighbor approximation by assigning the sampled datum to the nearest grid point and a finer grid (through reducing the grid spacing by a factor of $a \geq 2$) were used to minimize computation time and image error. The finer grid increased FOV in the second direction by a factor of a , with the central points corresponding to the desired image. The FDR algorithm was compared with the standard double-sized gridding algorithm ($W=4$, $\beta=18.5547$) in computer simulation and on MRI data. An analytical image, $m(x,y)=100(1-|x|)(1-|y|)$, was sampled with a spiral trajectory in k-space. The reconstructed images with both algorithms were compared to the analytical image for measuring the corresponding image error. MRI data were acquired from phantom and human coronaries using a gradient spiral sequence on a 1.5T scanner (GE Signa CV/i, Millwauki, WI). Imaging parameters were: 24 interleaves, 2685 pts/interleaf, $FOV=24 \times 24 \text{cm}^2$, $TE/TR/\text{flip}=1.1\text{ms}/11.7\text{ms}/30^\circ$, and sampling density compensation function from Hoge's (5). All computational activities were performed and timed on a PC (Intel P4, 2.4GHz CPU, 1.0GB RAM).

Results

The time and error measurements for both algorithms are summarized in Fig.1. For full k-space data set (Fig.1a), the reconstruction time of the FDR at $a=2$ was 766msec for image size 256^2 , slower than the gridding algorithm (406msec). For one-leaf data set (Fig.1b), the FDR at $a=2$ took 63 msec, faster than the gridding algorithm (125msec). The image errors of the FDR were less than 0.1% in all investigated image sizes (Fig.1c). The error of the FDR at $a=2$ was 0.026% at image size 256^2 , larger than that of the gridding algorithm (0.013%).

An example of reconstructed human coronary MRA images is illustrated in Fig.2. Both the gridding and FDR ($a=2 \& 8$) generated similar images.

Conclusions

The fast direct reconstruction (FDR) algorithm by combining the discrete and fast Fourier transforms is a viable method for reconstructing non-Cartesian k-space data. Compared to the standard gridding algorithm, the FDR can be a valuable alternative method with a penalty of modest increase in the whole image reconstruction time and image error, and a gain of higher real-time updating rate. The FDR algorithm at $a=2$ is suggested in practical applications.

References

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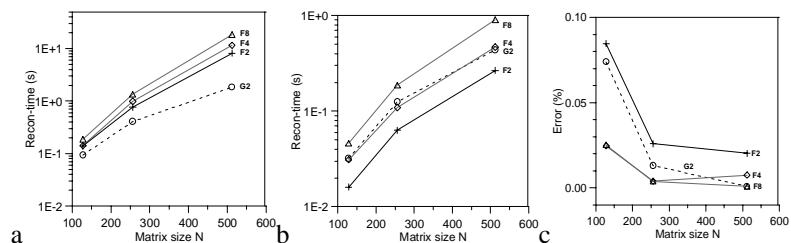


Fig.1. Reconstruction time for (a) full k-space data set and (b) one-leaf data set, and (c) image error. G2 is for the double-sized gridding and F2, F4, F8 for the FDR at $a=2$, 4, and 8, respectively.

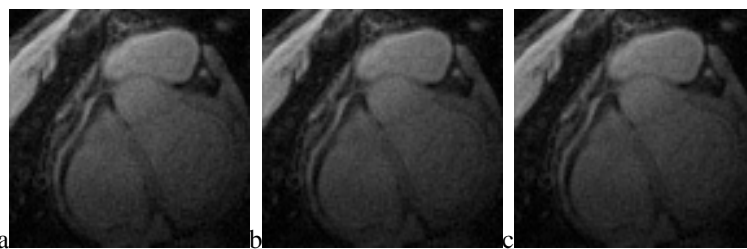


Fig.2. Coronary images reconstructed by double-sized gridding (a) and FDR with (b) $a=2$, (c) $a=8$. All three images demonstrate similar details of coronary arteries and surrounding structures.