Proper Normalization for the Gridding Algorithm

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Introduction

The gridding algorithm is routinely used in constructing images from data sampled on non-Cartesian trajectories (1). A normalization factor is required for the gridding algorithm to account for the discretization of the resampling convolution (2). This normalization factor depends on the data sampling trajectories. When constructing partial k-space data, which occurs in the sliding window reconstruction algorithm in time resolved imaging, care has to be taken to properly account for the normalization factor. In this paper, we review the gridding algorithm and point out the proper normalization for partial k-space data reconstruction.

Methods and Materials

The standard gridding algorithm (g1 in Table 1) includes five steps: sampling density compensation (*W*), convolution of the weighted data with a window function onto a Cartesian grid (*C*), normalization of the weighting at each grid point (*N*), fast Fourier transform (FFT) of the resampled data into image space, and roll-off correction (*c*) caused by the convolution (1,2). The normalization is used to reduce the convolution error during the resampling, $N = \sum_x C(x-x_0)W(x) = N(J)$, here x_0 is the grid and summation is over contributing points, which depends on the sampling trajectory (J). Four other variations to the standard gridding algorithm are tabulated in Table 1 for comparison to identify the effects of the normalization and data weighting and roll-off correction.

To assess the effects of normalization in constructing partial k-space data, sliding window reconstructing through updating only one interleaf or several interleaves in spiral imaging was investigated. Data were acquired on a 1.5T GE scanner with a gradient spiral sequence (FOV=24×24cm^2, 24 interleaves, 2685 pts/interleaf, and matrix size=256^2).

Table 1. Five variations of the gridding algorithm. (M = k-space data)

	Implementation	**Error (%)**
DFT	DFT$\{M \cdot W\}$	0.002
g1	c^{-1}FFT$\{N^{-1}(C*(M \cdot W))\}$	0.013
g2	c^{-1}FFT$\{C*(M \cdot W)\}$	5.593
g3	FFT$\{N^{-1}(C*(M \cdot W))\}$	0.461
g4	c^{-1}FFT$\{N^{-1}(C*M)\}$	0.700
g5	c^{-1}FFT$\{C*M\}$	17.111

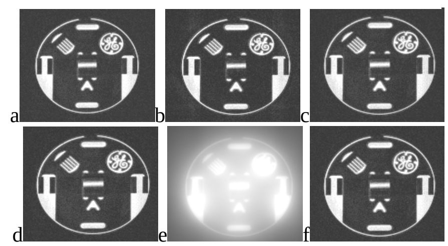

Fig.1. Reconstructions of the gridding algorithm (a-e: g1, g2, g3, g4, g5) and the DFT algorithm (f).

Results & Discussion

For an analytical image, the image errors (RMS between the reconstructed images and the original) in all five implementations of the gridding algorithm, as well as the discrete Fourier transform (DFT) method, are listed in Table 1. The normalization was essential to reduce image error to be < 1% (g1, g3, & g4 in Table 1). The three implementations with normalization (Fig.1a,c,d) generated similar image quality close to that of the DFT (Fig.1f). Streaking artifacts were noticeable when the normalization was taken out (Fig.1b). The implementation without normalization and sampling density compensation generated the worst image (Fig.1e).

For partial k-space data reconstruction, the normalization based on the sampling trajectory leads to substantial image errors, which increased as the data size (# of interleaves) decreased (Table 2 & Fig.2). The implementation without normalization (g2) had only an image error constant over data size (Table 2). This is because the normalization factor based on partial k-space trajectory is not proper for the final image, which requires a normalization factor based upon the full trajectory.

Table 2. Error between sum & complete images.

Implementation	**# of interleaves**	**Error (%)**
g1	1	44.64
	6	1.37
	8	0.40
	12	0.04
g2	1, 6, 8, 12	5.59

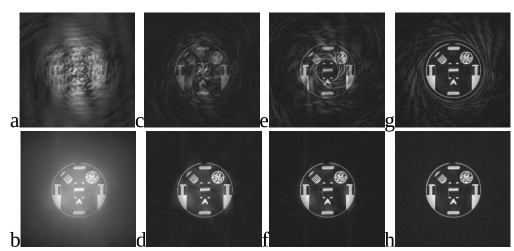

Fig. 2. Intermediate (top) and sum (bottom) images illustrate the effects of normalization based on partial trajectories: (a, b) 1-leaf, (c, d) 6-leaves, (e, f) 8-leaves, and (g, h) 12-leaves.

Conclusion

The standard gridding algorithm with normalization, density weighting and roll-off correction is an accurate reconstruction method for full k-space data sets. It should be noted that the normalization has to be based upon the full k-space trajectory. Accordingly, in reconstructing partial k-space data, a normalization based on the full trajectory, not the partial trajectory, should be employed.

References

1. Jackson JI, et al. IEEE TMI 1991;10:473-478.
2. Meyer et al. MRM 1992; 28:202-213.