Efficient Three-Dimensional Reconstruction for Non-Cartesian Acquisitions

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Introduction: Non-Cartesian and non-uniformly sampled image acquisitions continue to grow in popularity as the computing power necessary to efficiently reconstruct these data sets has become widely available. Three-dimensional (3D) acquisition methods can also be designed that utilize non-uniform sampling. In both 2D and 3D cases, fast reconstruction methods based on the Fast Fourier Transform (FFT) still require interpolation of the sampled data onto a uniformly spaced grid. Gridding reconstruction [1] has been most widely used for this task but requires computation of a density compensation function (DCF) to account for possibly large variations in sampling density. An accurate DCF for 3D data sets can be difficult to define and time-consuming to compute.

Block Uniform Resampling (BURS) [2] and its more stable variations [3,4] use interpolation matrices computed with Singular Value Decomposition (SVD) to eliminate the need for a DCF. While the SVD is also time-consuming to calculate, it can be "pre-computed" and stored for a given sampling trajectory, to be used for subsequent reconstructions. Thus, this computation time is reduced to a one-time cost. The purpose of this work was to investigate the feasibility and efficiency of performing complete 3D reconstruction with a modified BURS algorithm using pre-computed SVD results.

Methods: Acquisition of 3D data utilized the cone-stack hybrid trajectory [5]. This spiral-based sampling strategy covers 3D *k*-space by beginning along the surfaces of a cone and then flattening out into a planar spiral, as shown in Figure 1. This permits radial acquisition of points near the *k*-space origin while also supporting anisotropic or "thin-slab" volumes. The slope of the cone section is varied to fill out *k*-space and cones are acquired in pairs in $\pm k_z$. The acquired data has a widely varying sampling density, being heavily oversampled near the *k*-space origin while approaching uniform density in the outer regions. The trajectory was implemented on a GE 1.5 T Signa CV/i scanner and phantom data was acquired to assess the performance of the reconstruction algorithm.

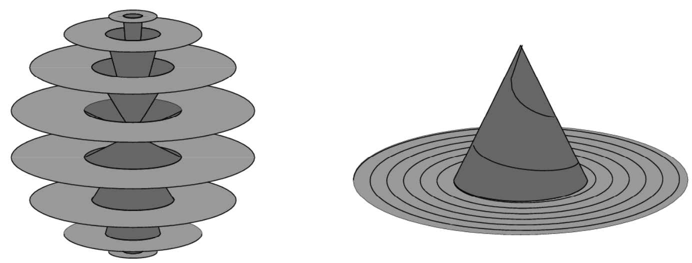

Figure 1: Spiral samples begin on the surface of a cone in the center of 3D *k*-space, and then transition to parallel planes to yield a "hat" shape. Spirals are interleaved on the hat surfaces, and the shapes are varied and nested to provide complete coverage in 3D *k*-space.

Interpolation coefficients were pre-computed for the designed cone-stack trajectory for each point in the Cartesian grid similar to [4]. A matrix of interpolation coefficients **A** is computed using all points κ_i within $\delta\kappa$ of each non-uniformly sampled point and points k_j within a neighborhood of Δk in the Cartesian grid as $A_{ij} = \mathrm{sinc}(|\kappa_i - k_j|)$. Using SVD, blocks of points in the interpolation matrix are computed from $\mathbf{A}^{\#} = (A^T A + \rho I)^{-1} A^T$ (where ρ is a regularization parameter) and stored. For image reconstruction, the pre-computed interpolation coefficients are read in and used to create the Cartesian-interpolated data followed by the 3D FFT for final volume image generation. Thus, the time-consuming SVD calculations are done only once for all acquisitions using the sampling trajectory. The algorithm was implemented in MATLAB.

Results: The implemented cone-stack trajectory acquired a 20 x 20 x 5 cm^3 field-of-view with 0.8 x 0.8 x 4 mm^3 resolution and required a total of 270 spiral interleaves utilizing a 14 ms readout time for a total of 483840 sampled points. Neighborhood sizes of $\delta\kappa = 2$, $\delta\kappa = 4$, and $\delta\kappa = 6$ in the k_x-k_y plane and $\delta\kappa = 1$ along k_z were tested along with $\Delta k = 1$ (inclusive) in the Cartesian grid of 256 x 256 x 12. Calculation and storage of the interpolation matrices took approximately 3 hours and 1 GB of storage for this 3D trajectory, which need only be performed once. Utilizing the stored information, full volume reconstructions required approximately 2 minutes (with a significant fraction of that time used for file reading). Sample phantom images from the 3D data set are shown in the figure below.

Discussion: Efficient reconstruction of 3D non-Cartesian data sets with highly non-uniform sampling densities has been demonstrated to generate high-quality volume reconstructions without computing a DCF. By pre-computing interpolation data that depends only on the sampling pattern, large 3D volumes can be rapidly reconstructed and viewed by trading computation time for storage space, making them more suitable for use in a clinical environment. Limitations of this technique include the assumption of a fixed *k*-space trajectory, meaning that corrections for irregularities in the *k*-space sampling pattern (arising from off-resonance effects or eddy currents) may not be possible. Extending similar storage strategies for 3D imaging to other non-uniform sampling reconstruction methods (such as NUFFT [6]) may be useful for future improvements in 3D imaging.

References:

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Figure 2: Sample slices from a 256 x 256 x 12 voxel data set reconstructed with the modified BURS algorithm using pre-computed interpolation coefficients. The image on the left uses a neighborhood size of $\delta\kappa = 2$, the center image has neighborhood size $\delta\kappa = 4$, and the image on the right uses $\delta\kappa = 6$. Each image set required 2 minutes to reconstruct using the stored interpolation data. As can be seen, all three images show very good image quality from this high signal-to-noise ratio data